

Les « points de bascule » dans l'histoire de notre Planète

Tipping points and Earth's history

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Le pape François éreinte les climatosceptiques

(www.lemonde.fr, 4 octobre 2023)



« Je me rends compte au fil du temps, écrit-il, que nos réactions sont insuffisantes alors que le monde qui nous accueille s'effrite et s'approche peut-être d'un point de rupture. »

« Nous avons beau essayer de les nier, de les cacher, de les dissimuler ou de les relativiser, poursuit-il, les signes du changement climatique sont là, toujours plus évidents.

Tipping points and Earth's history

1 – Introduction

- Earth's history : a succession of tipping points
- What is climate ?
- Climate surprises (IPCC 2001 – DOs and HEs)

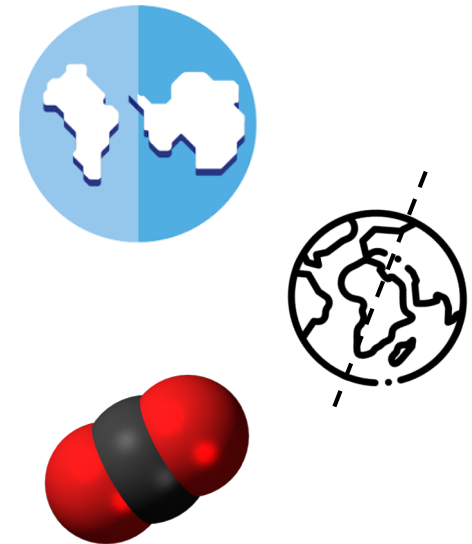
2 – Ice ages and astronomy

- Some history
- Stochastic resonance vs. quasi-linear models
- Deglaciations as a tipping point

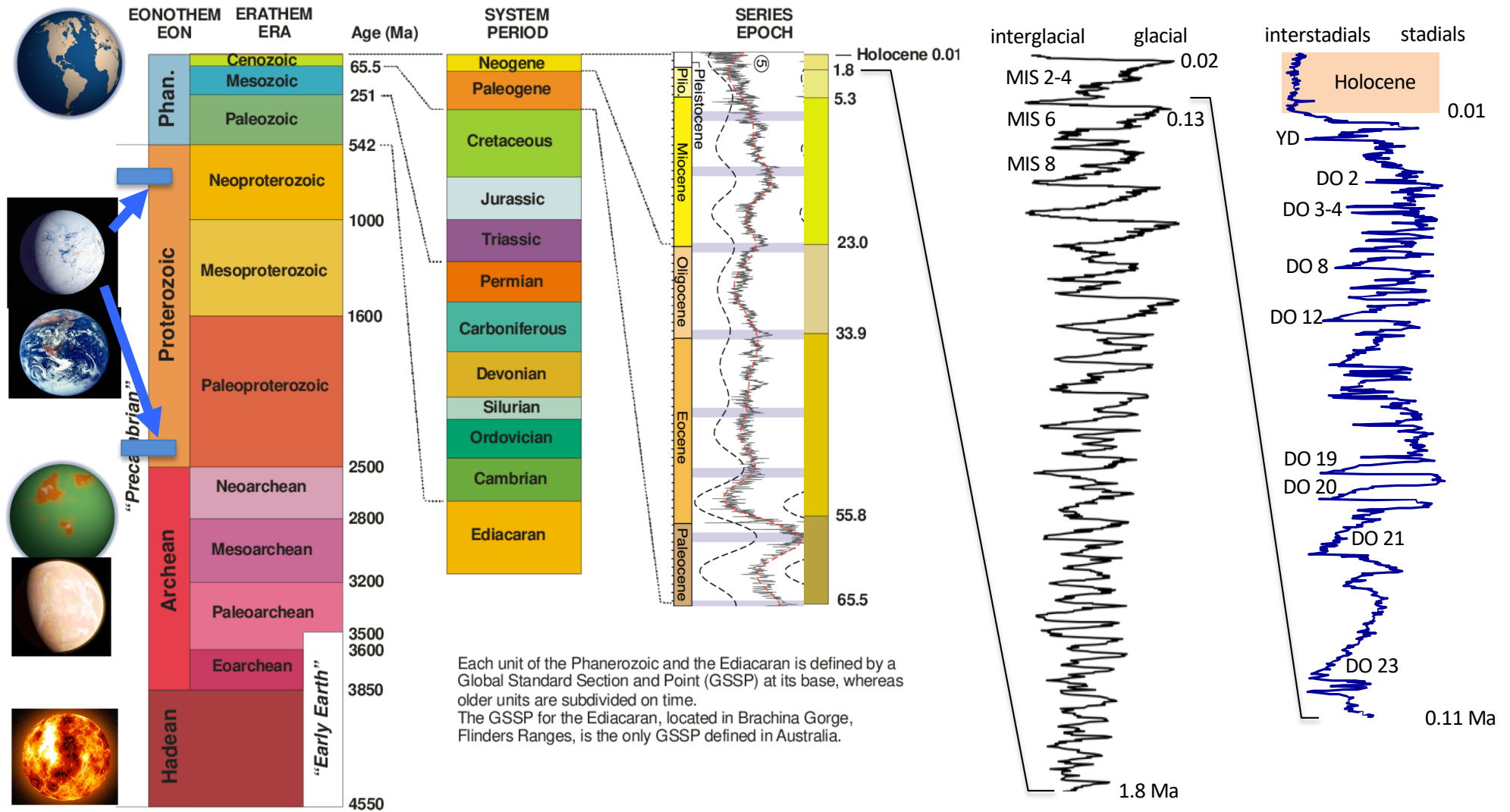
3 – Carbon and astronomy

- Some data
- A new theory (involving a tipping point!)

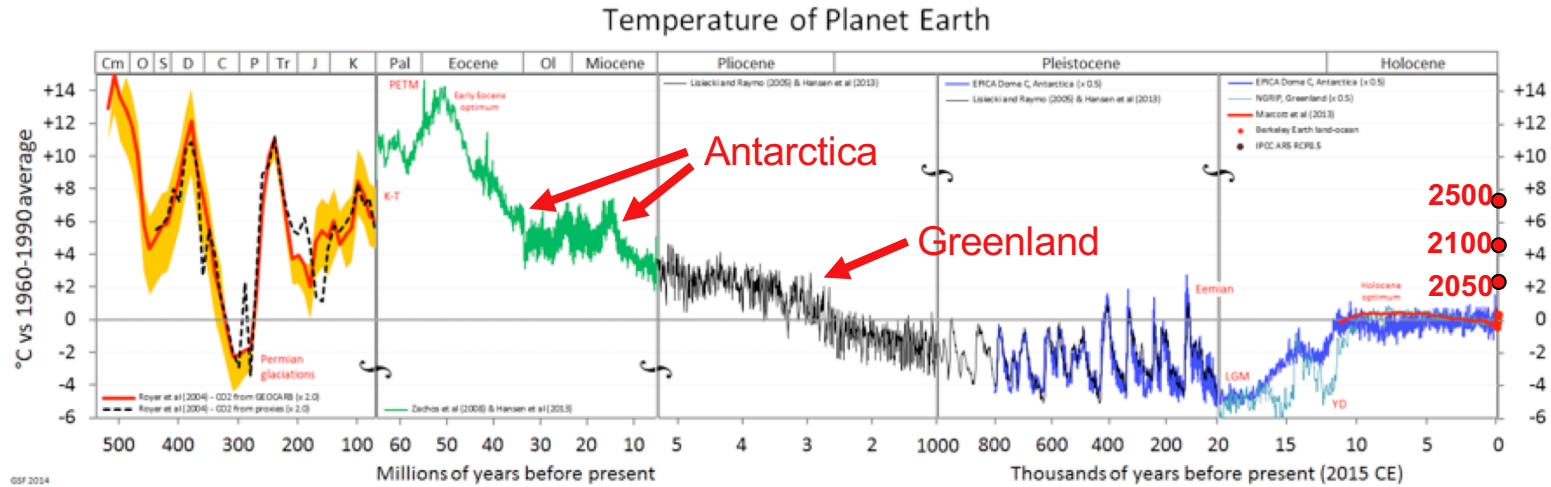
4 – Concluding remarks



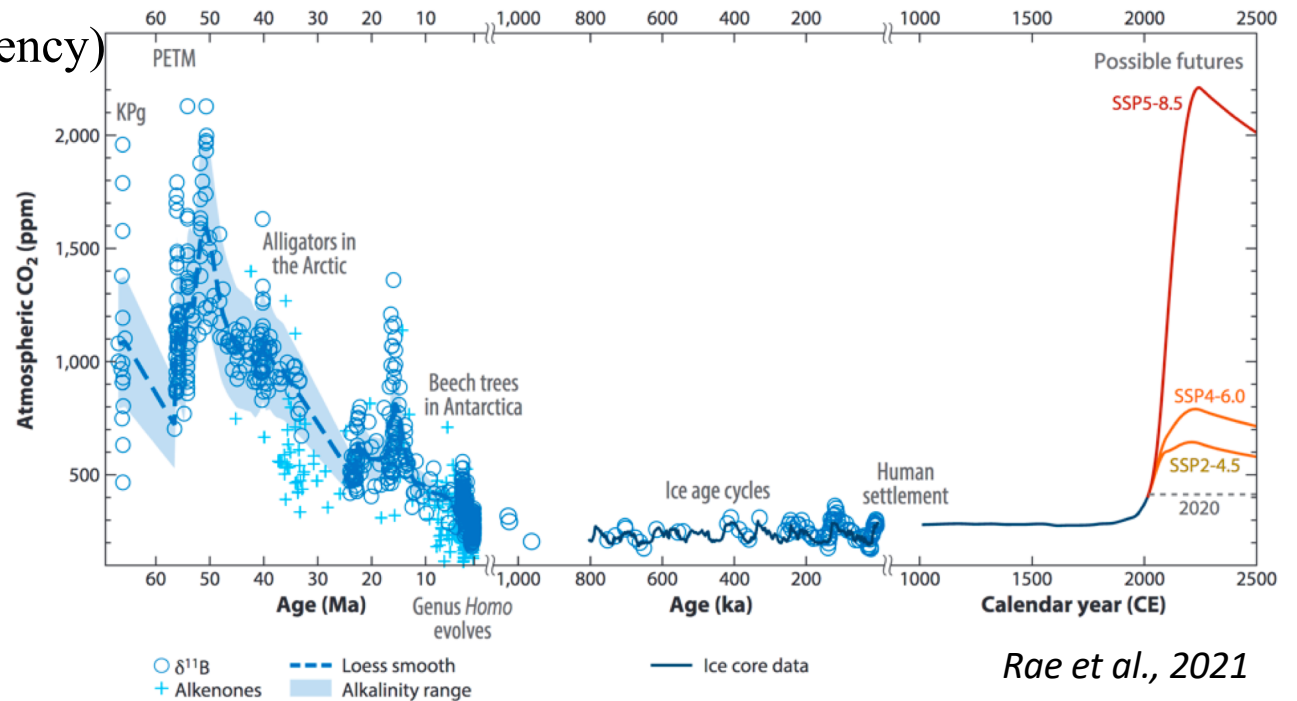
Earth's history : a succession of tipping points



Period – one word **but** several meanings



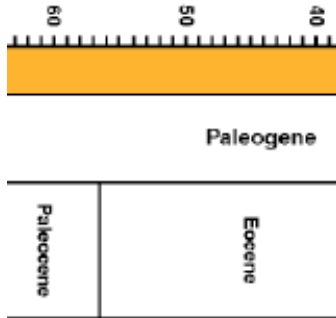
- old or recent period (time)
- short or long periodicity (frequency)
- return period (probability)
- abruptness



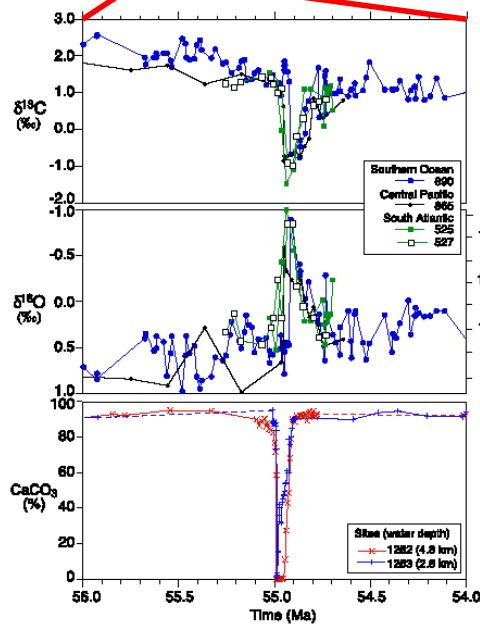
Rae et al., 2021

What is a tipping point ?

Paleocene-Eocene transition



- extinction de nombreux foraminifères benthiques
- renouvellement des mammifères terrestres, notamment: diversification des primates



Injection massive de carbone

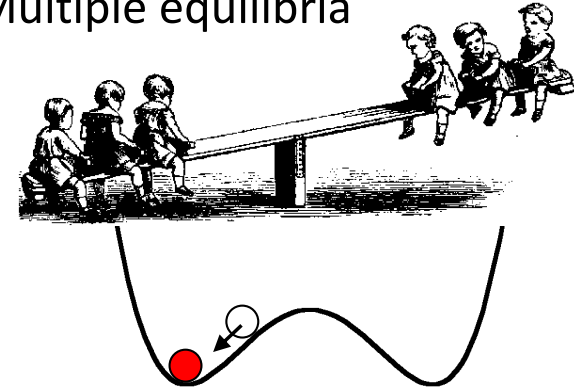
Réchauffement « brutal »

Acidification de l'océan

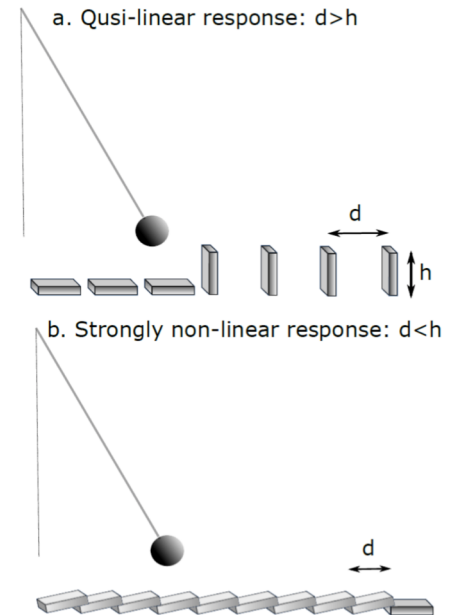
irréversible

réversible (200ka)

Multiple equilibria



Multiple tipping-points : domino effect

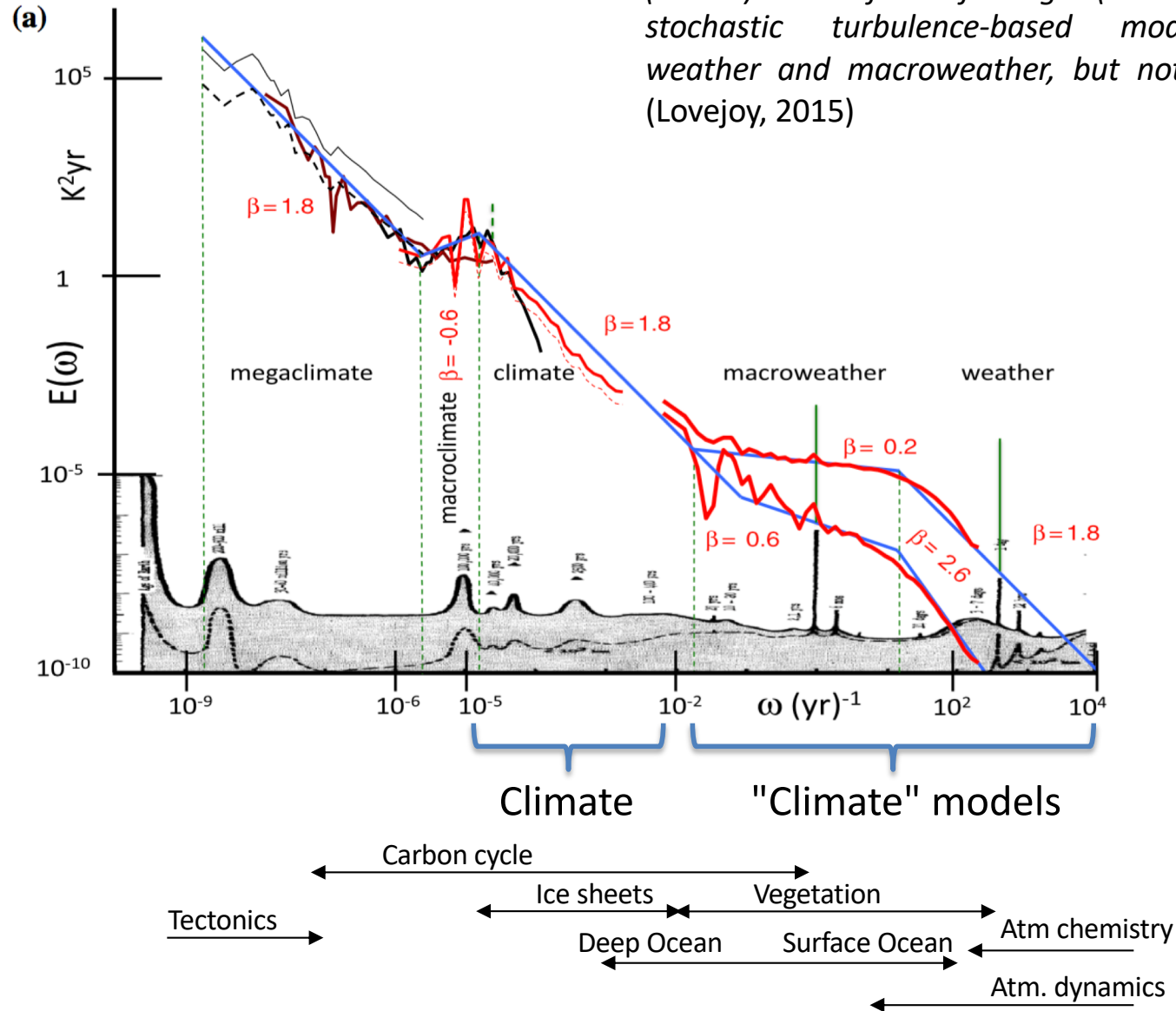


What is climate ?

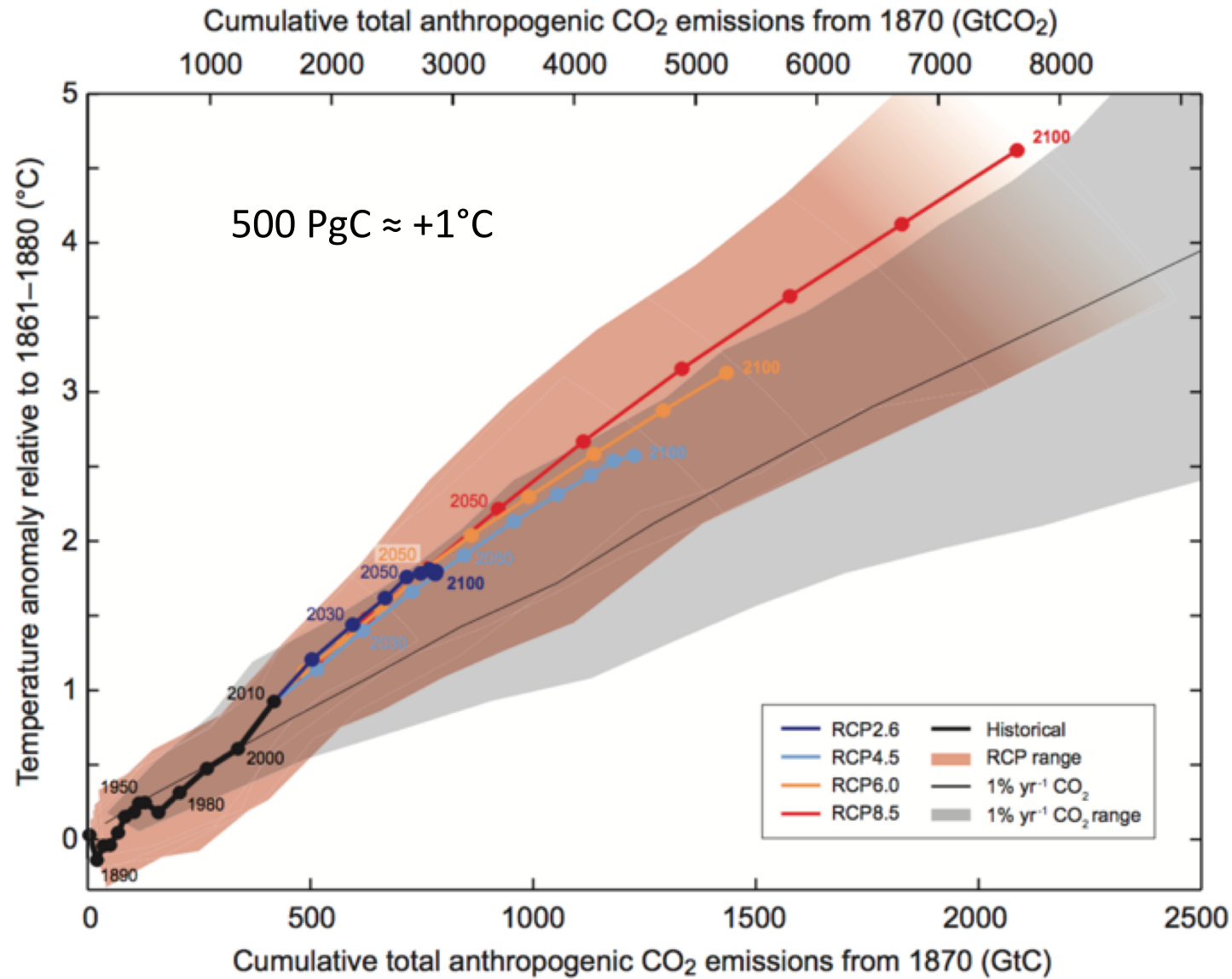
Lovejoy (2015):

Based on the spectral slope of T

« Both deterministic General Circulation Models (GCM's) with fixed forcings ("control runs") and stochastic turbulence-based models reproduce weather and macroweather, but not the climate. »
(Lovejoy, 2015)

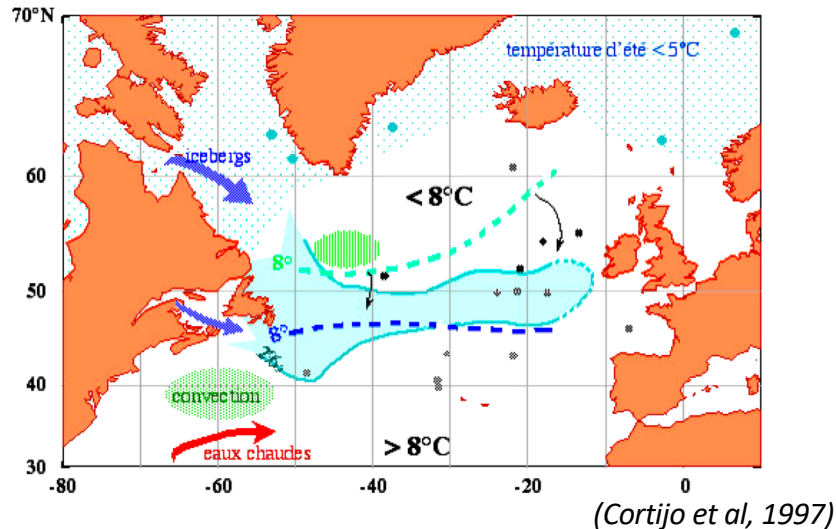
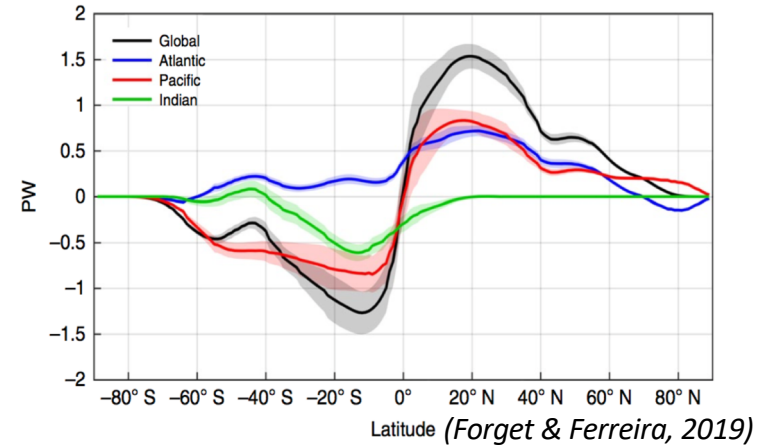
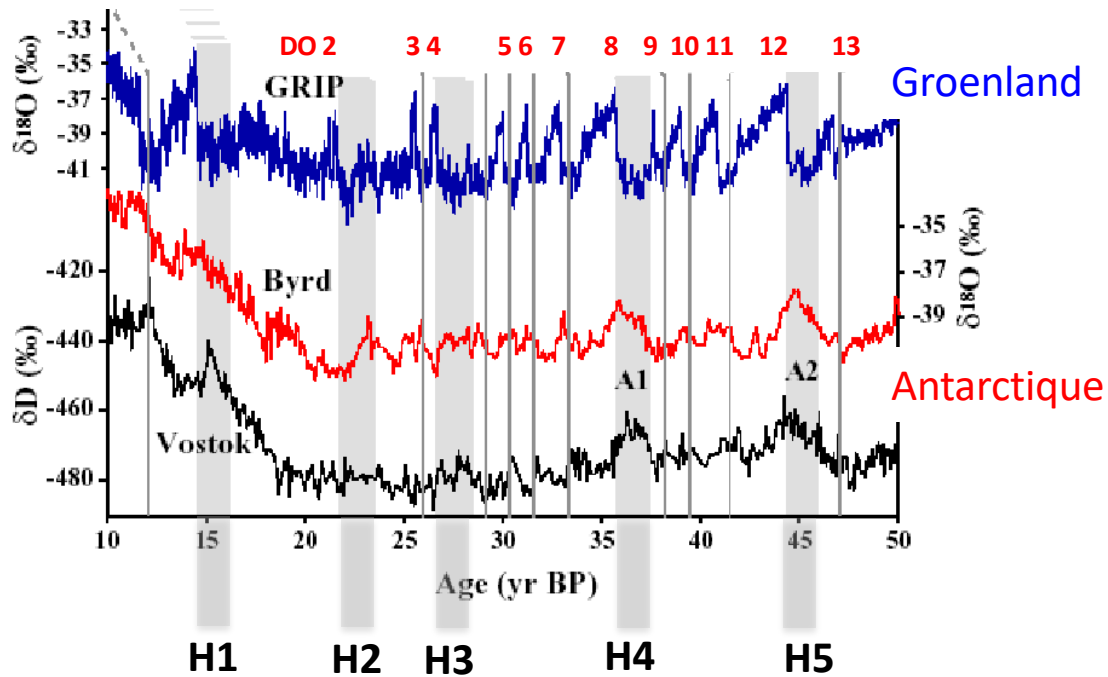


Climate change (IPCC)



(IPCC 2014)

Dansgaard-Oeschger and Heinrich events



250 μ m sédiment glaciaire de l'Atlantique Nord



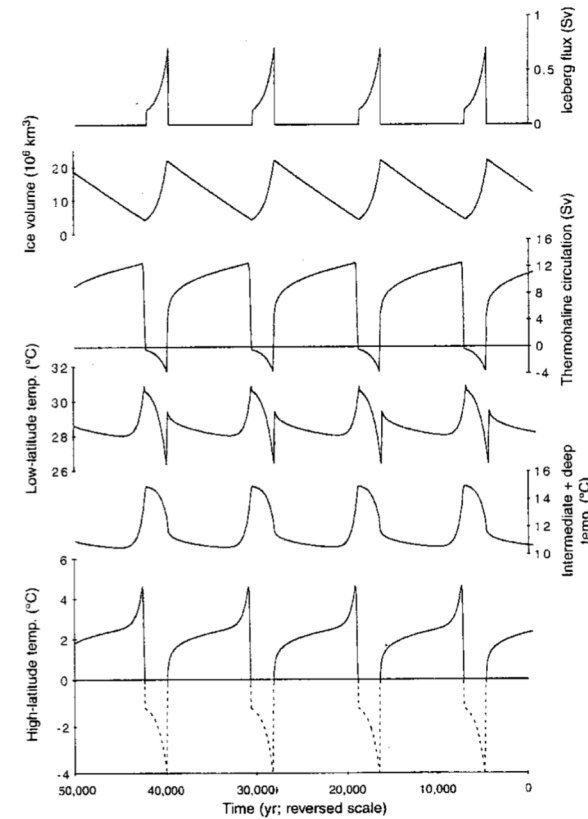
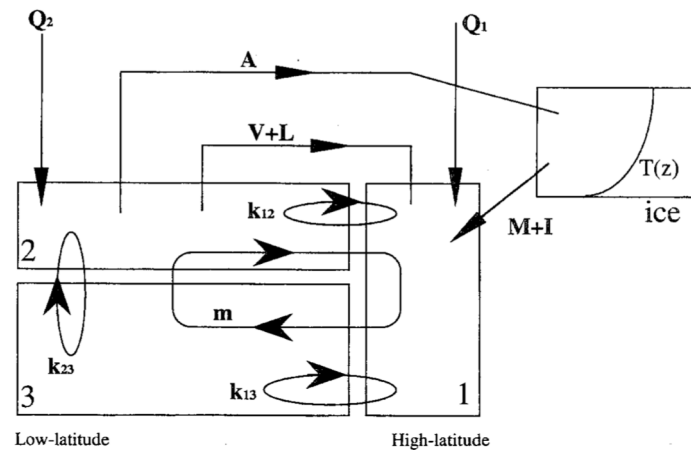
sédiment des niveaux de Heinrich



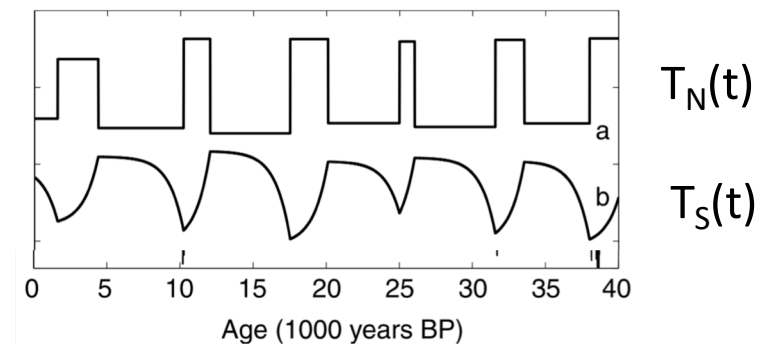
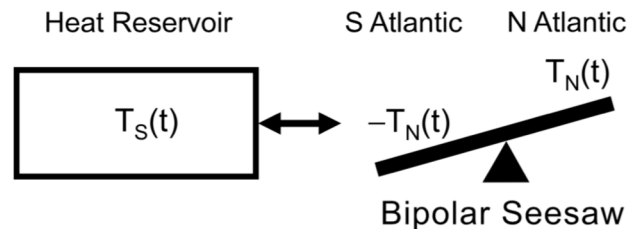
Dansgaard-Oeschger and Heinrich events

Paillard and Labeyrie (Nature 1994)

Coupling a bimodal ocean model (Stommel, 1961) with an oscillating ice-sheet model (MacAyeal, 1993)



Stocker and Johnsen (Paleoceanography 2003)



Tipping points

IPCC (2001):

The rapid forcing of a non-linear system has a high prospect of producing surprises.

The climate system involves many processes and feedbacks that interact in complex non-linear ways. This interaction can give rise to thresholds in the climate system that can be crossed if the system is perturbed sufficiently. **There is evidence from polar ice cores** suggesting that atmospheric regimes can change within a few years and that large-scale hemispheric changes can evolve as fast as a few decades. For example, the possibility of a threshold for a rapid transition of the Atlantic THC to a collapsed state has been demonstrated with a hierarchy of models.

Changes in vegetation, through either direct anthropogenic deforestation or those caused by global warming, could occur rapidly and could induce further climate change. It is supposed that the **rapid creation of the Sahara about 5,500 years ago represents an example** of such a non-linear change in land cover.

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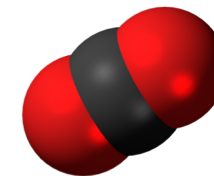
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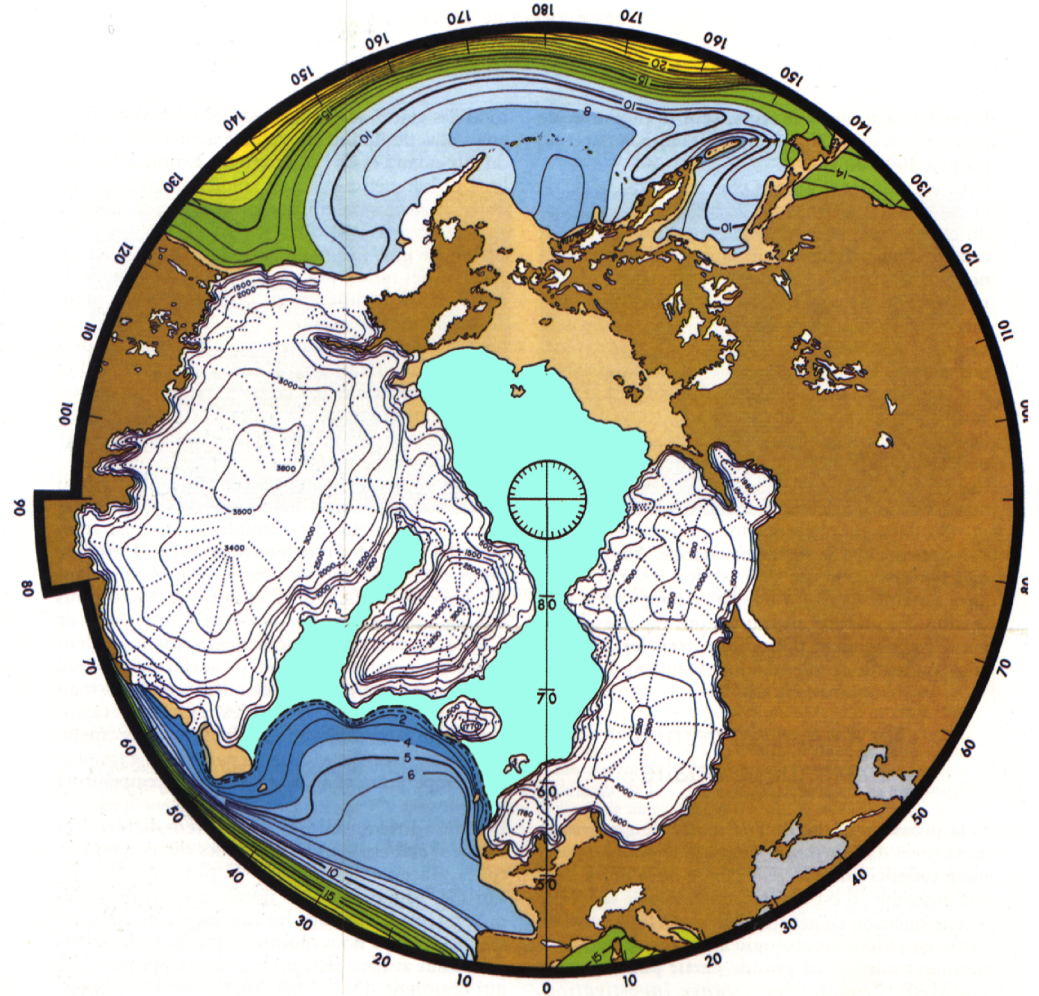
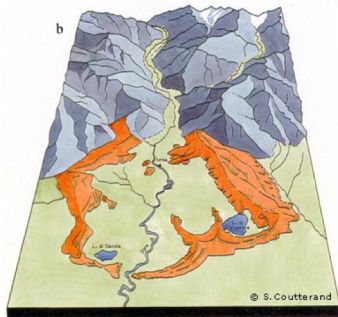
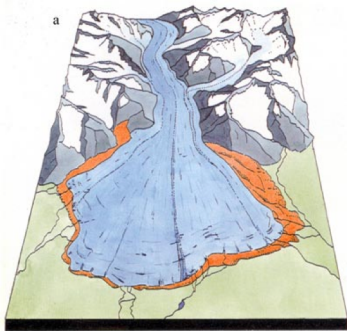
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4 – Concluding remarks

Ice ages – the archetypal climatic change



Two theories of climate change since the 19th century

(J. Fourier, 1824)

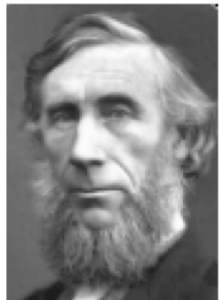
CO₂

Astronomy

(3rd movement of the Earth, Hipparque, 127 BC)

Ebelmen, 1845

RÉVOLUTIONS
DE LA MER.
PAR J. ADHÉMAR,
CHIMISTE ET ASTRONOME.
PARIS.
GABRIAN GOREY et V. DALMONT,
Libraires, quai des Augustins, 39 et 41.
BACHELIER, Libraire, quai des Augustins, 35.
MATHIAS, Libraire, quai Napoléon, 35.
1842



Tyndall,
1861, 1863



Arrhenius,
1896



Ekholm, 1901



Milanković,
1920, 1941



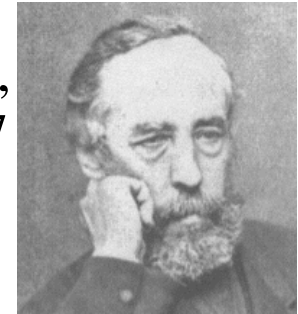
Chamberlin,
1897, 1899



Callendar,
1938



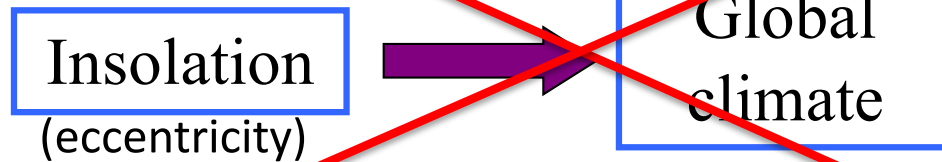
Calder,
1974



Croll,
1864, 1867

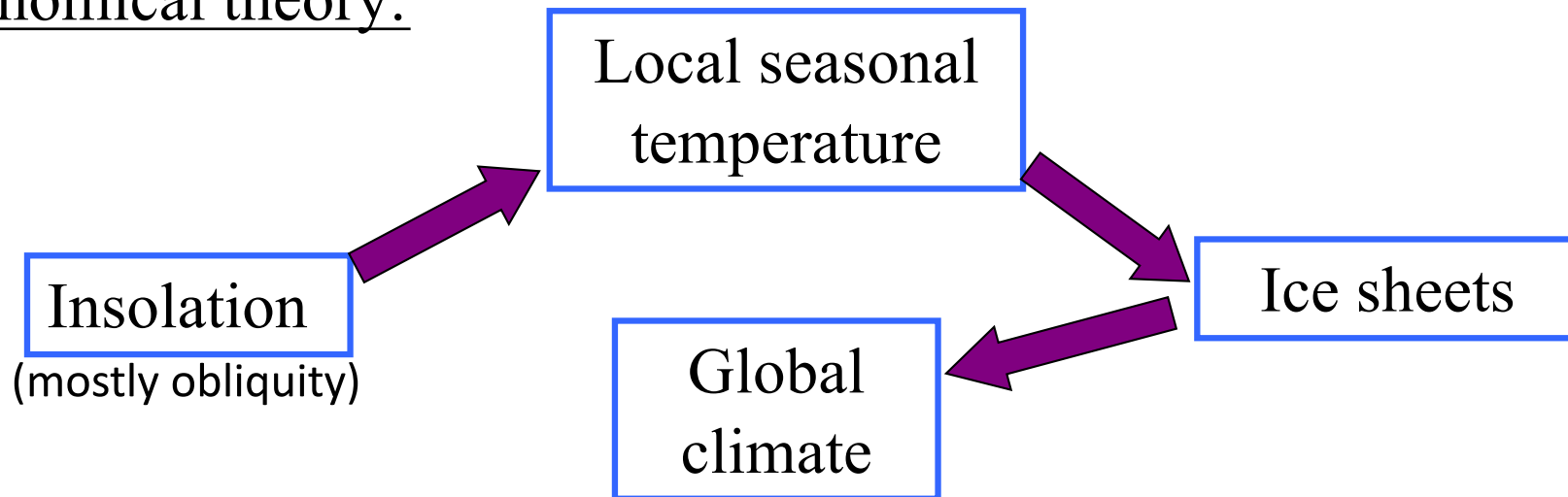
Causality of ice ages

“Naïve theory”:



(John Herschel, 1830)

Astronomical theory:



Geochemical theory (CO₂):



THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[FIFTH SERIES.]

APRIL 1896.

XXXI. *On the Influence of Carbonic Acid in the Air upon the Temperature of the Ground.* By Prof. SVANTE ARRHENIUS *.



Arrhenius, 1896

I should certainly not have undertaken these tedious calculations if an extraordinary interest had not been connected with them. In the Physical Society of Stockholm there have been occasionally very lively discussions on the probable causes of the Ice Age; and these discussions have, in my opinion, led to the conclusion that there exists as yet no satisfactory hypothesis that could explain how the climatic conditions of that age could be realized in so short a time as that which

$$\begin{aligned} \text{CO}_2(\text{LGM}) / \text{CO}_2(\text{PreInd}) &= 180 \text{ ppm} / 280 \text{ ppm} \\ &= 0.64 \end{aligned}$$

Latitude	0.67xCO ₂						2.0xCO ₂					
	Dec.-Feb.	March-May.	June-Aug.	Sept.-Nov.	Mean of the year.	Mean of the year.	Dec.-Feb.	March-May.	June-Aug.	Sept.-Nov.	Mean of the year.	Mean of the year.
70	-29	-30	-34	-31	-31	33	60	61	60	61	60	61
60	-30	-32	-34	-33	-32	34	61	61	58	61	60	61
50	-32	-33	-33	-34	-33	37	61	61	55	60	59	60
40	-34	-34	-32	-33	-32	37	60	58	54	56	57	57
30	-33	-32	-31	-31	-31	35	56	54	50	52	53	53
20	-31	-31	-30	-31	-30	35	52	50	49	50	50	50
10	-31	-30	-30	-30	-30	32	50	49	49	49	49	49
0	-30	-30	-31	-30	-30	31	49	49	50	49	49	49
-10	-31	-31	-32	-31	-31	32	50	50	51	50	50	50
-20	-31	-32	-33	-32	-32	32	52	52	53	52	52	52
-30	-33	-33	-34	-34	-33	34	55	56	58	56	56	56
-40	-34	-34	-33	-34	-33	36	58	60	60	60	60	60
-50	-32	-33	-	-	-	38	60	61	-	-	-	-
-60	-	-	-	-	-	-	-	-	-	-	-	-

-3°C

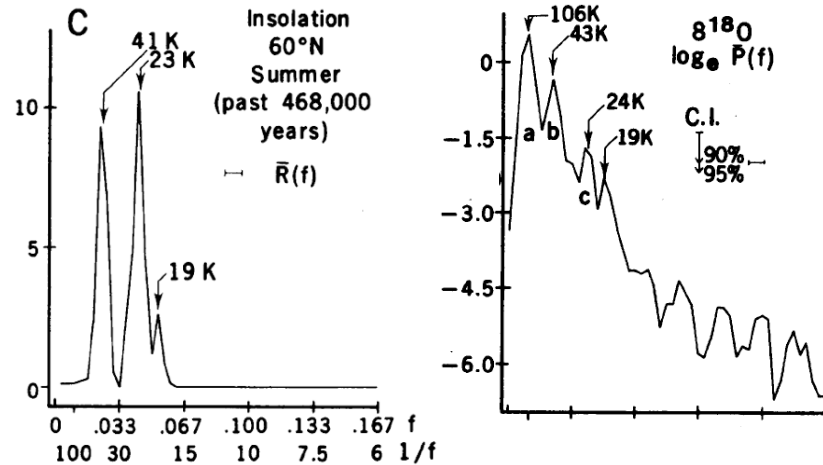
+5°C

Prof. S. Arrhenius on the Influence of Carbonic Acid

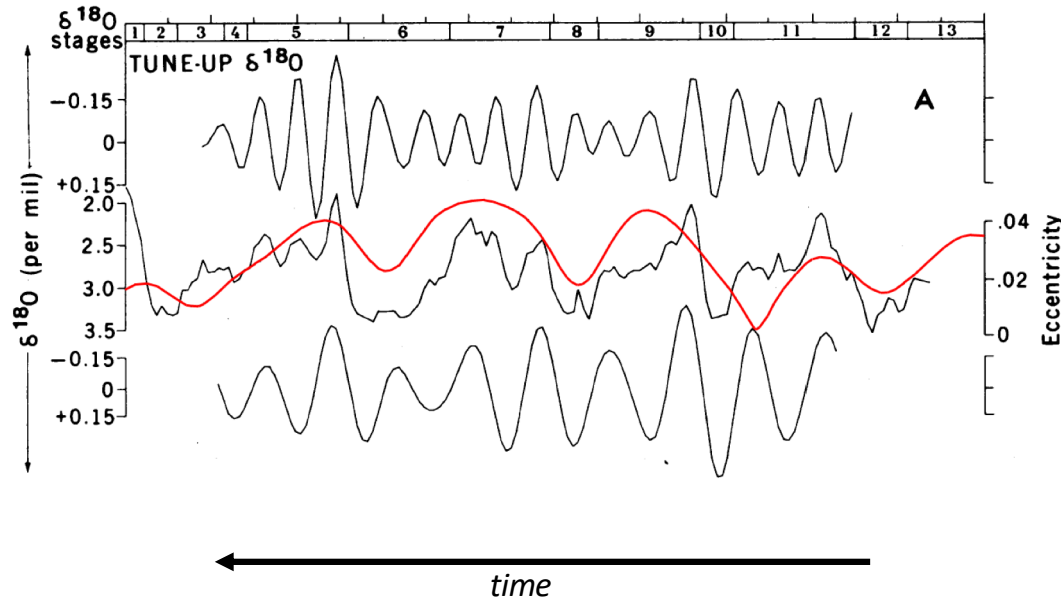
In order to get the temperature of the ice age between the 40th and 50th parallels, the carbonic acid in the air should sink to 0.62 - 0.55 of its present value (lowering of temperature 4°-5° C.). The demands of the geologists, that at the genial

Hays et al., 1976

Variations in the earth's orbit: **pacemakers** of the ice ages



1 – A clear signature of astronomical periodicities (19ka, 23ka, 41ka)



2 – The dominant cycle (100ka) is not linked directly to the forcing, but « **in some way** » to changes in eccentricity.

the 100,000-year climate cycle is driven in some way by changes in orbital eccentricity. As before, we avoid the obligation of identifying the physical mechanism of this response, and instead characterize the behavior of the system only in general terms. Specifically, we abandon the assumption of linearity

Benzi et al., 1982

Stochastic resonance

In this section we shall present the model used here to study the effect of changes in the annually averaged solar radiation on the global earth temperature T . Our starting point is the usual, deterministic energy-balance model

$$C \frac{dT}{dt} = R_{in}(T) - R_{out}(T). \quad (1)$$

Here C is the thermal capacity of the earth, R_{in} is the incoming solar radiation and R_{out} the outgoing radiation. The parameterizations for R_{in} and R_{out} are:

$$R_{in}(T) = Q\mu \quad (2a)$$

$$R_{out}(T) = \alpha(T)Q\mu + \varepsilon(T) \quad (2b)$$

where $\varepsilon(T)$ is the long-wave surface radiation, $\alpha(T)$ the globally averaged albedo and Q is a long period average of incoming solar radiation. The dimensionless parameter μ will allow us to introduce an explicit variation in the solar input.

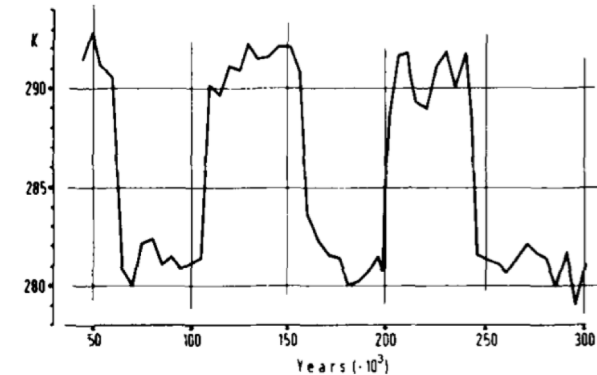
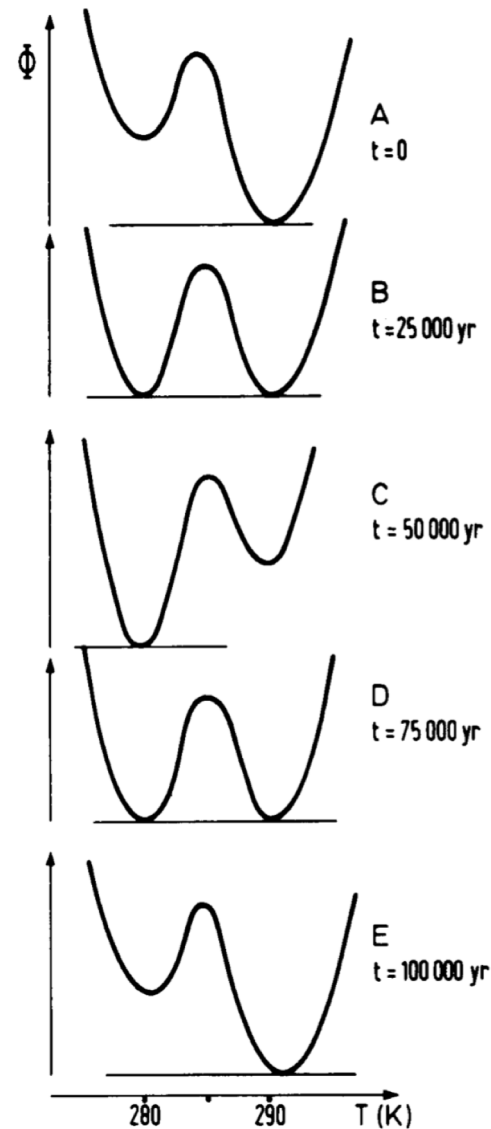


Fig. 4. Computer simulation of eq. (5) for heat-budget model with two observable climates at 280 and 290 K. The variance of the noise was about $0.15 \text{ K}^2/\text{year}$.

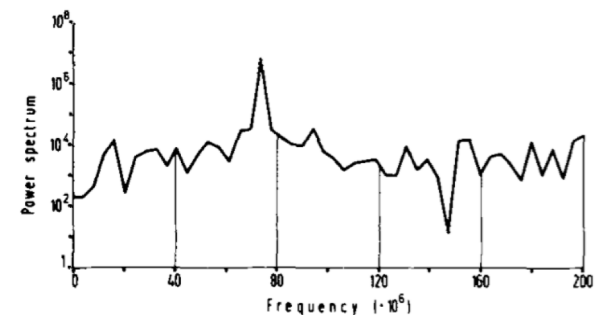
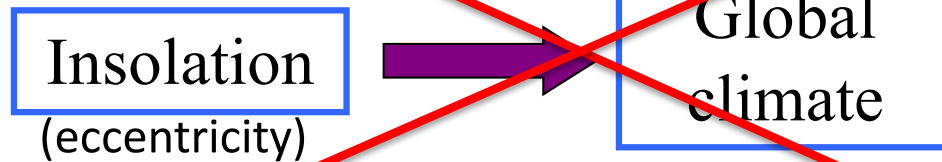


Fig. 5. The power spectrum of the solution shown in Fig. 4.

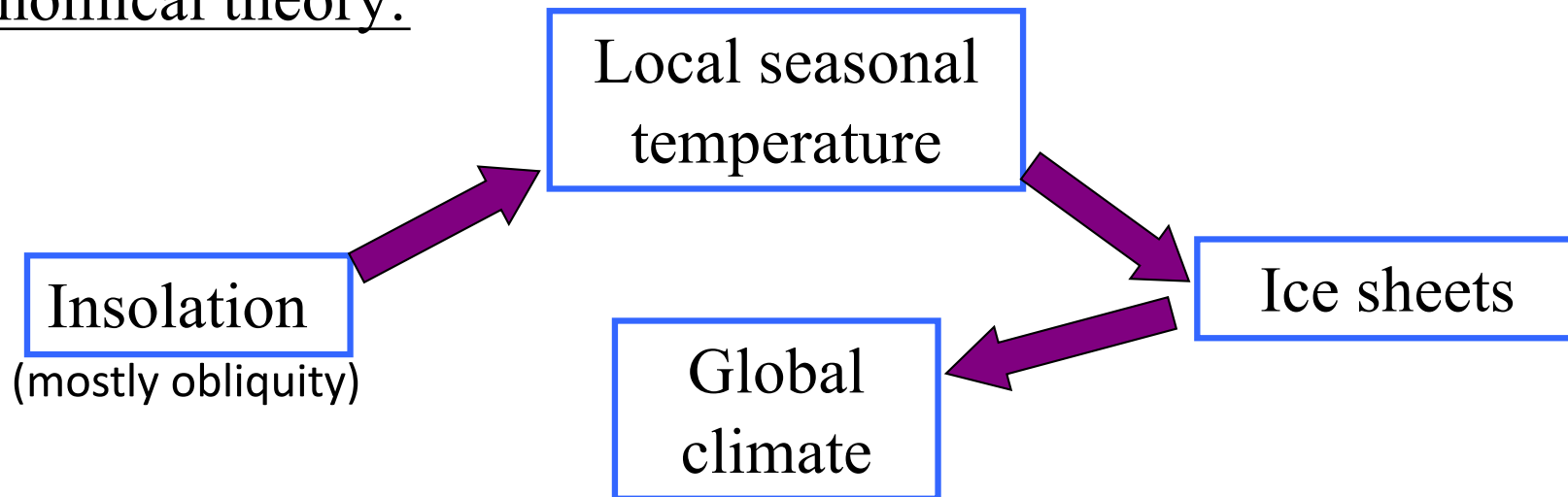
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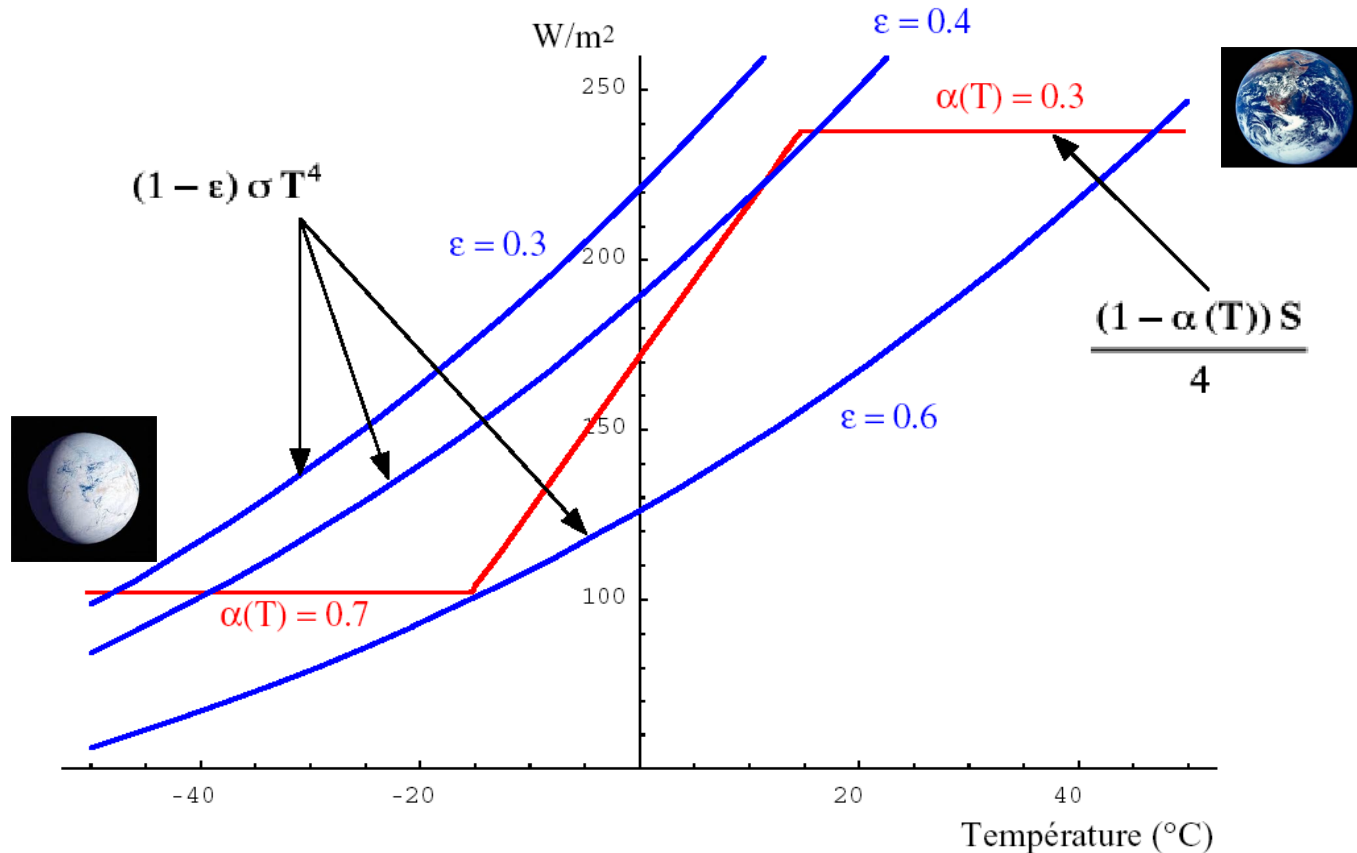


Rétroaction glace - albédo

On introduit empiriquement un coefficient ε = effet de serre:

$$(1-\alpha) S/4 = \sigma T_E^4 = (1-\varepsilon) \sigma T^4 \quad \text{avec } \varepsilon = 0,4$$

On suppose que l'albédo est grand ($\alpha = 0,7$) quand la Terre est gelée; petit ($\alpha = 0,3$) quand elle est bleue:



Calder (*Nature*, 1974)

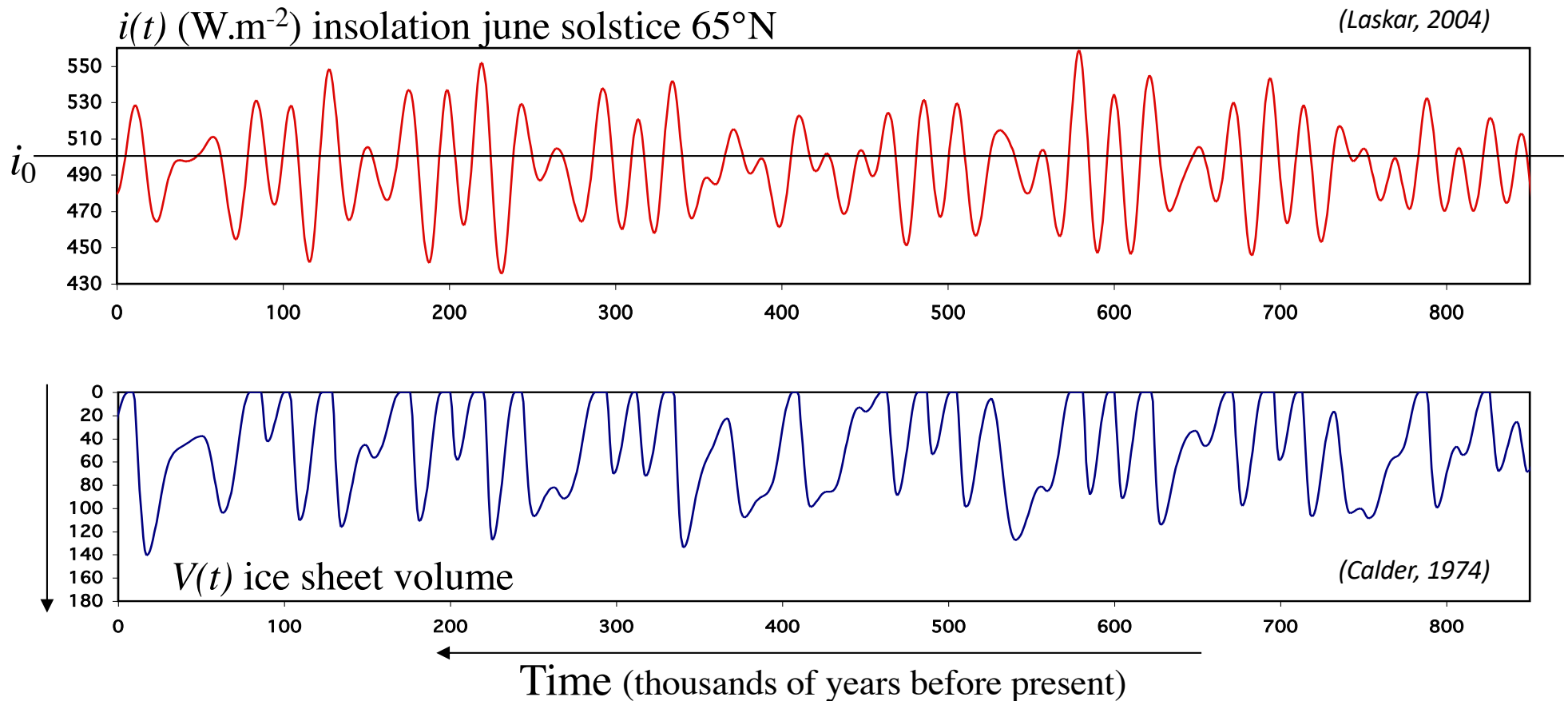
$$\frac{dV}{dt} = -k(i(t) - i_0)$$

$$V(t) \geq 0$$

$$k = \begin{cases} k_M, & \text{if } i(t) > i_0 \\ k_A, & \text{if } i(t) < i_0 \end{cases}$$

$$\begin{cases} i_0 = 502 \text{ W}\cdot\text{m}^{-2} \\ \frac{k_A}{k_M} = 0,22 \end{cases}$$

« Meteorological processes are so notoriously nonlinear that my assumptions are almost frivolous »



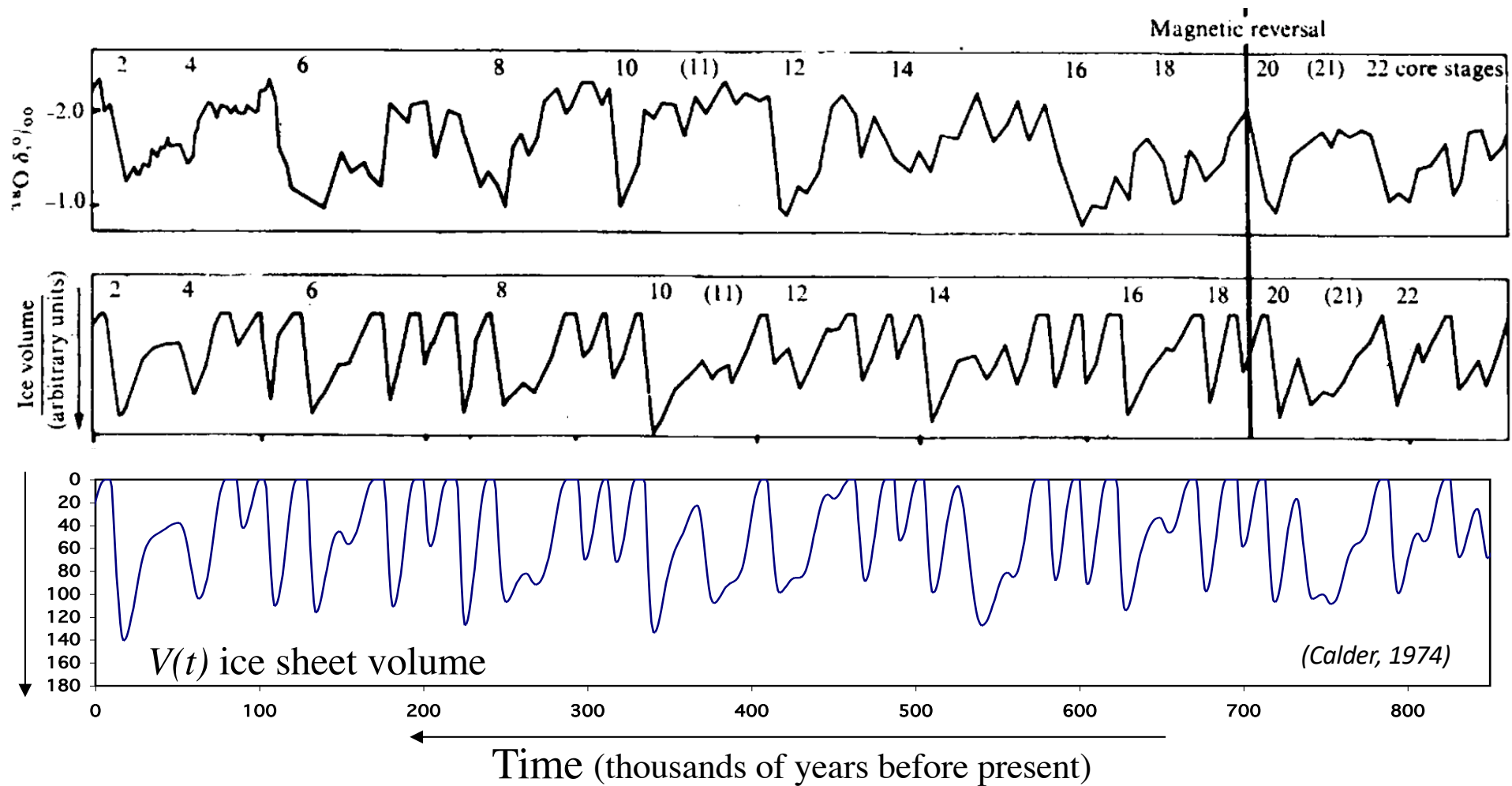
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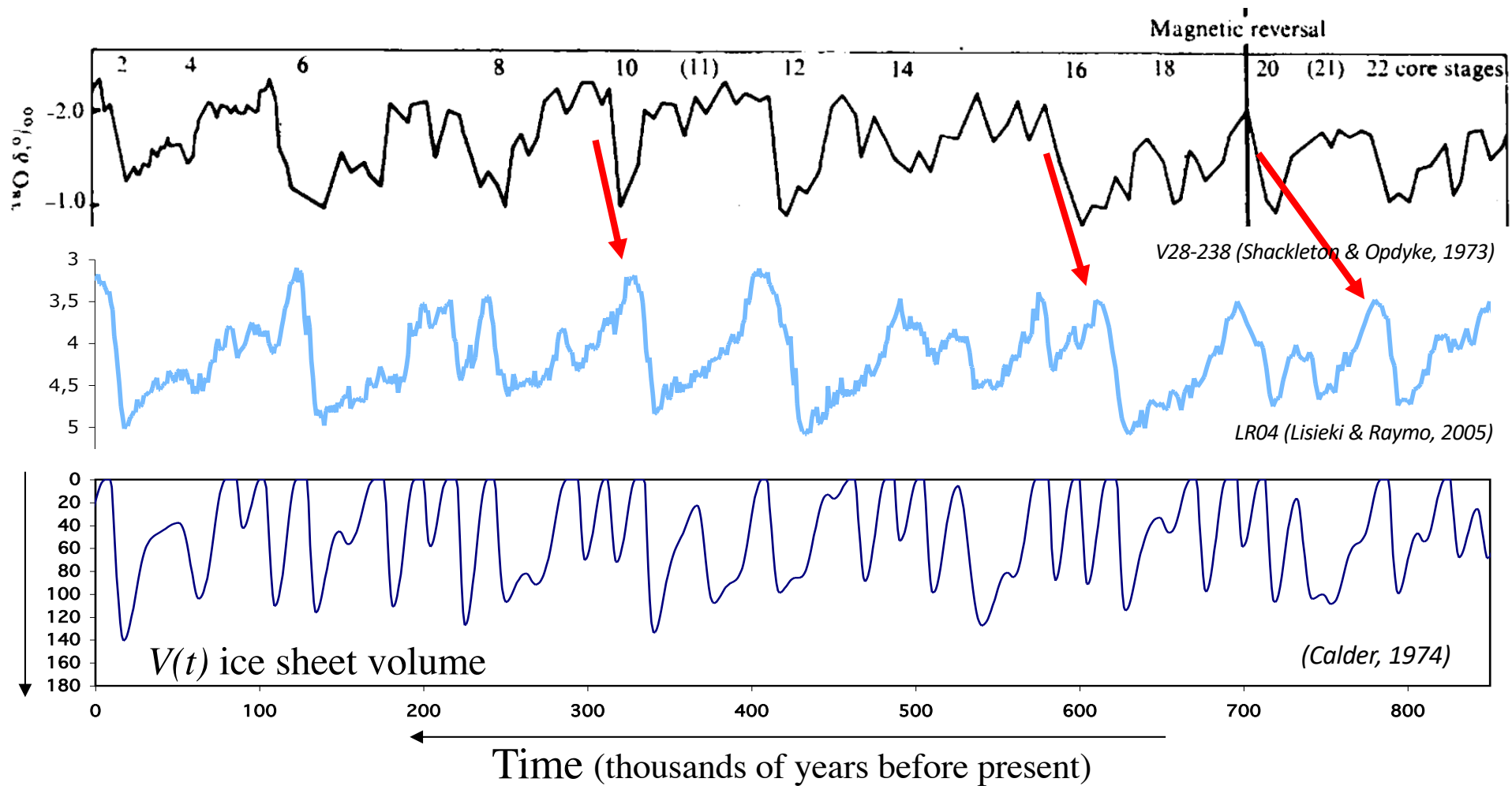
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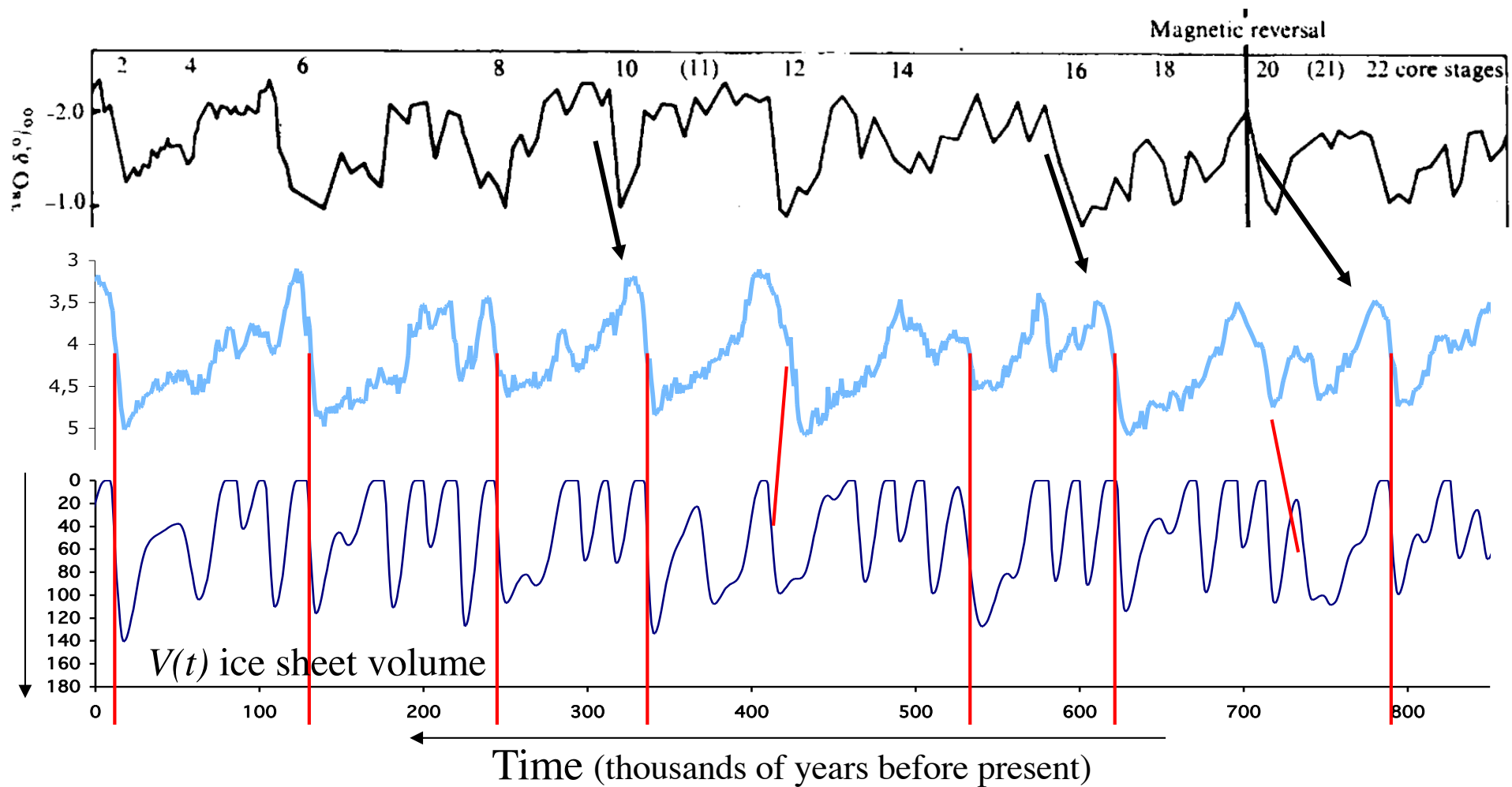
Calder (*Nature*, 1974)

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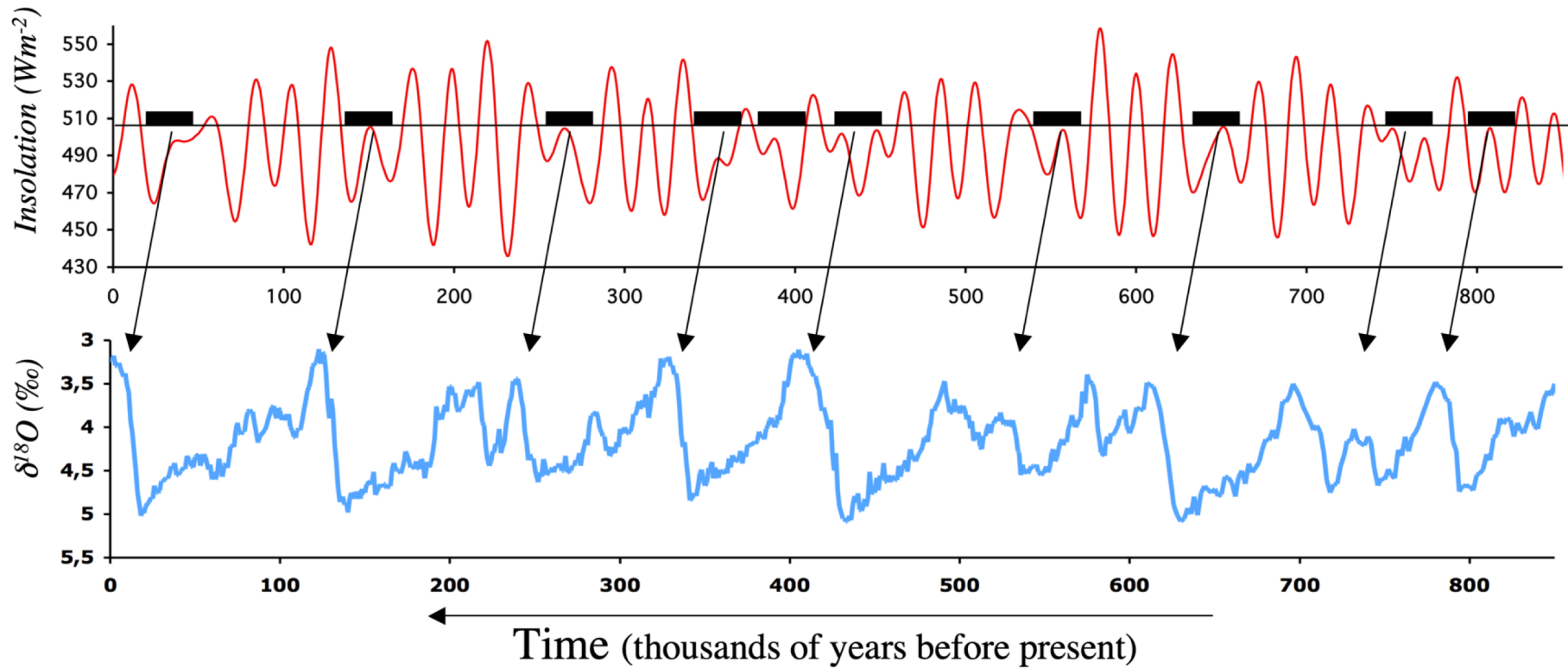
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The 100-kyr cycle

is arising from the amplitude modulation of precession by the eccentricity

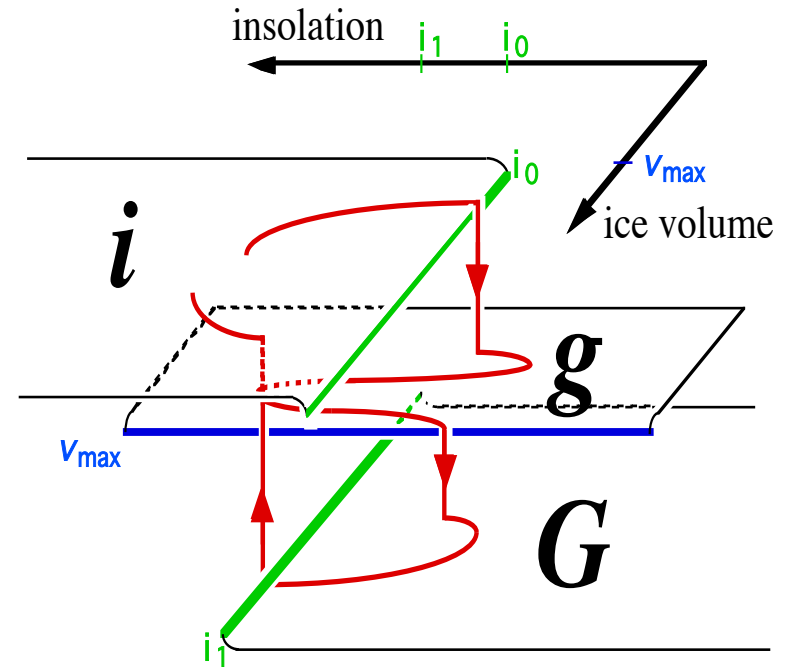
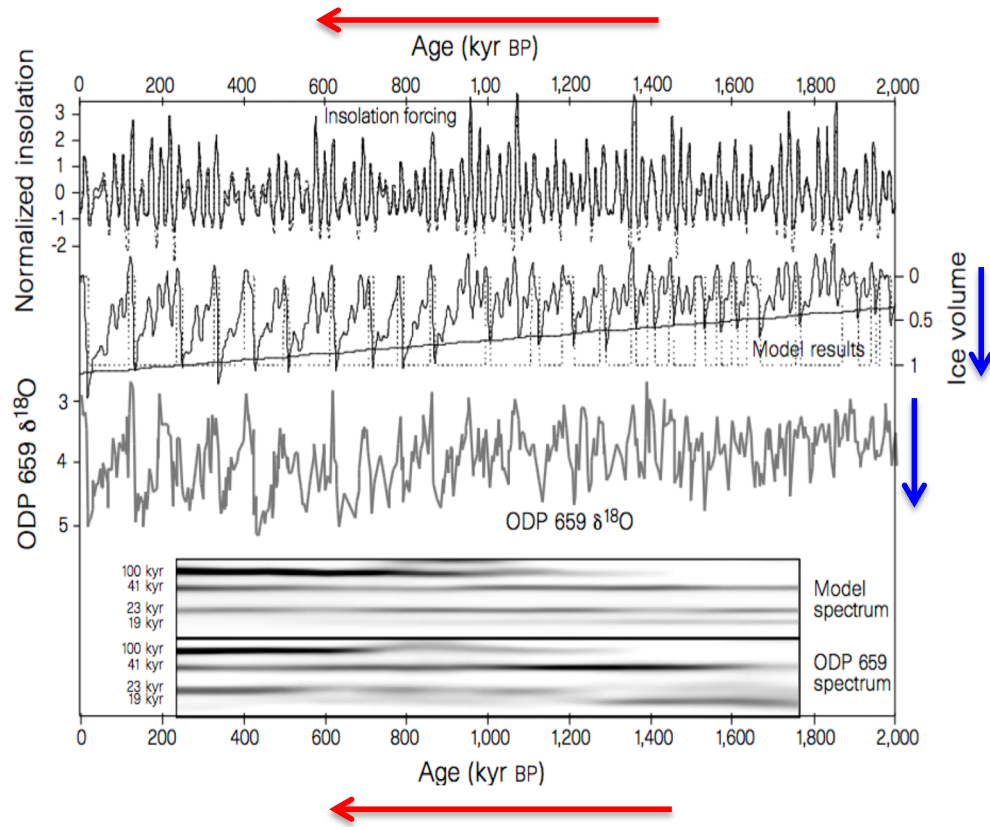


Low eccentricity \rightarrow small insolation maxima \rightarrow larger ice sheet \rightarrow termination

The ice age problem

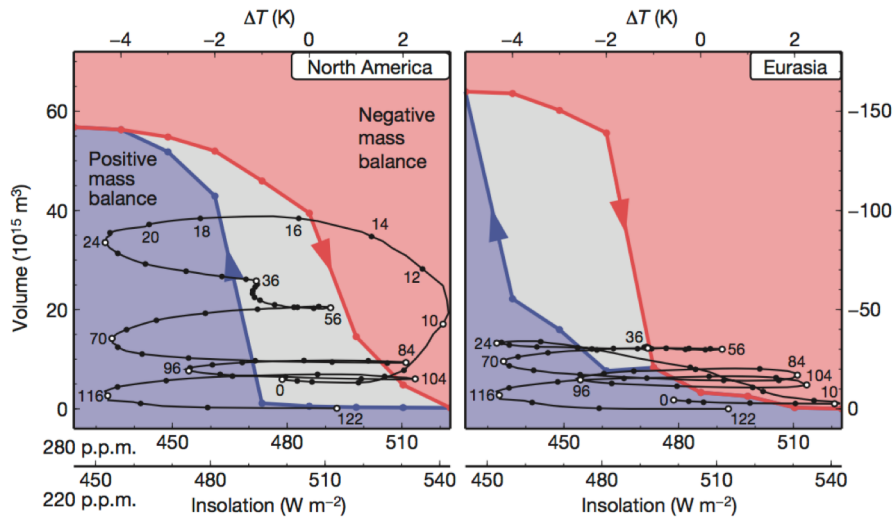
Thresholds and hysteresis can explain :

- the 100,000 yr cycles
- the Mid-Pleistocene Transition
- the shape of the oscillations
- ...



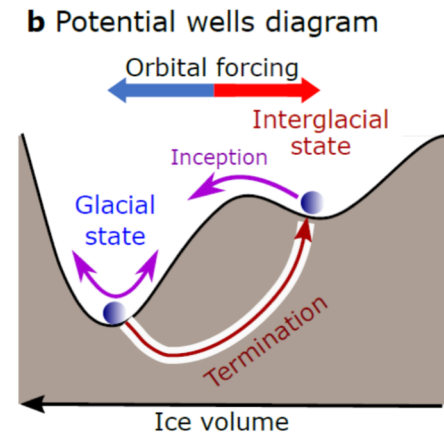
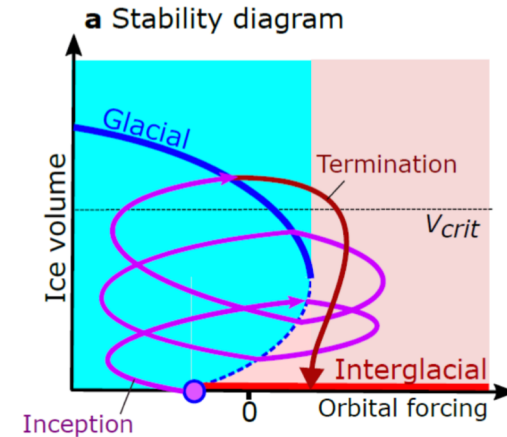
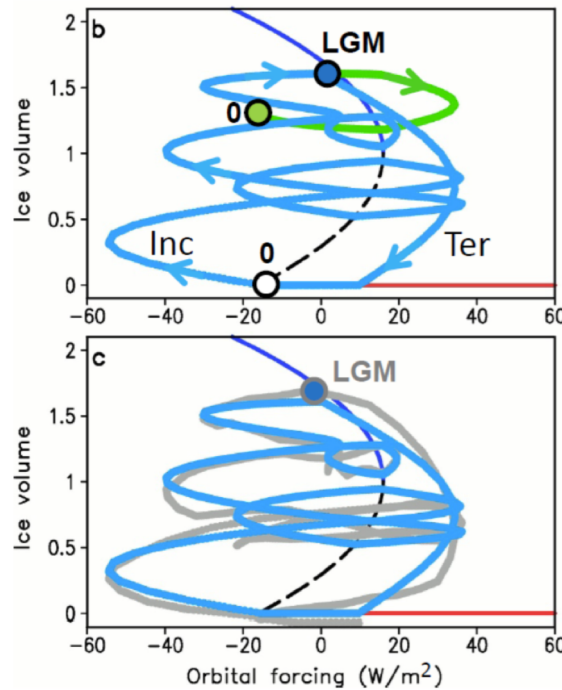
Paillard (1998)

The ice age problem

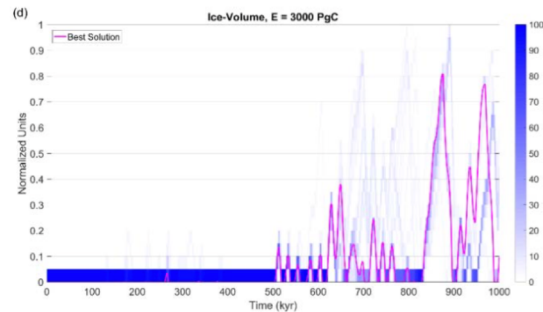
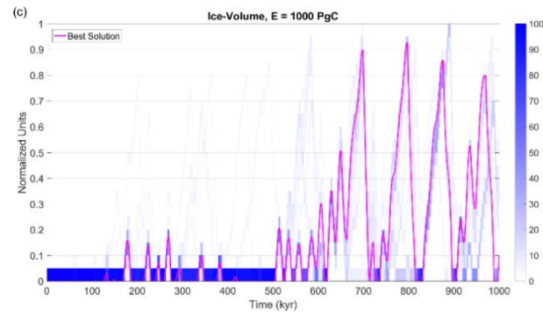
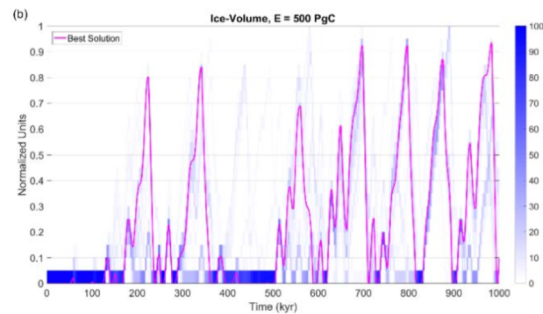
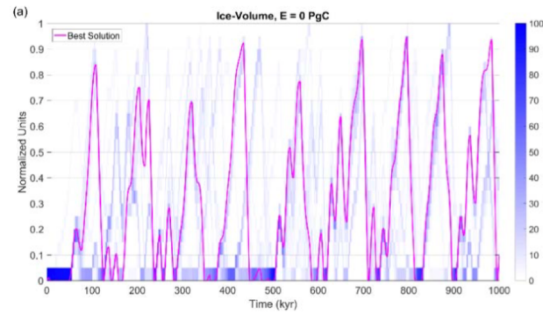
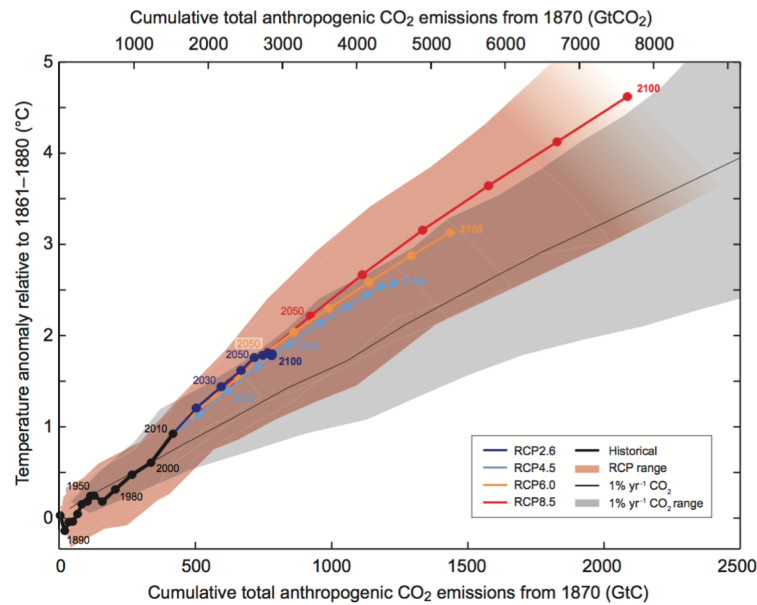
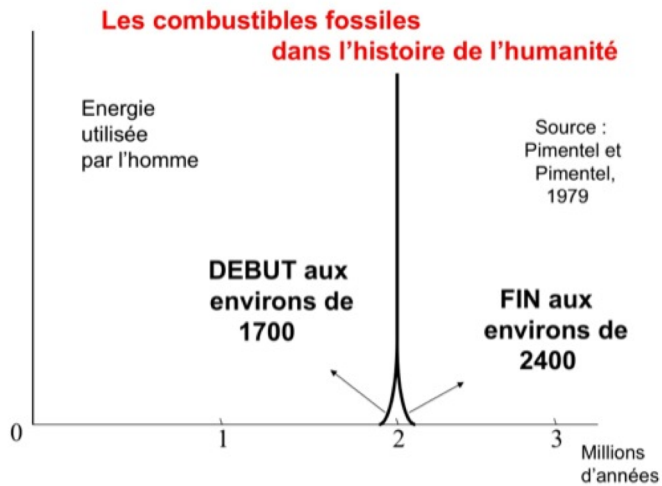


Abe-Ouchi et al. (Nature 2013)

Ganopolski (CPD 2023)



Durée de l'Anthropocène = temps de relaxation du carbone ?



Prochaine glaciation dans:

0 GtC => 50 kans

500 GtC => 180 kans

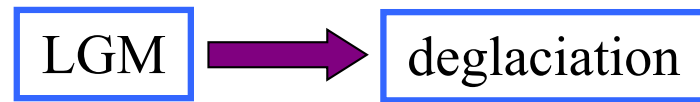
1000 GtC => 600 kans

3000 GtC => 820 kans

CO₂ as a trigger for deglaciations

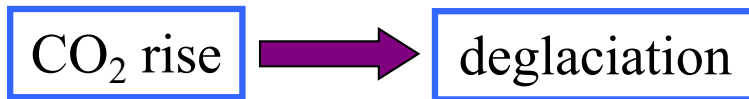
The key (robust) result:

**Deglaciations are « triggered »
by glacial maxima.**



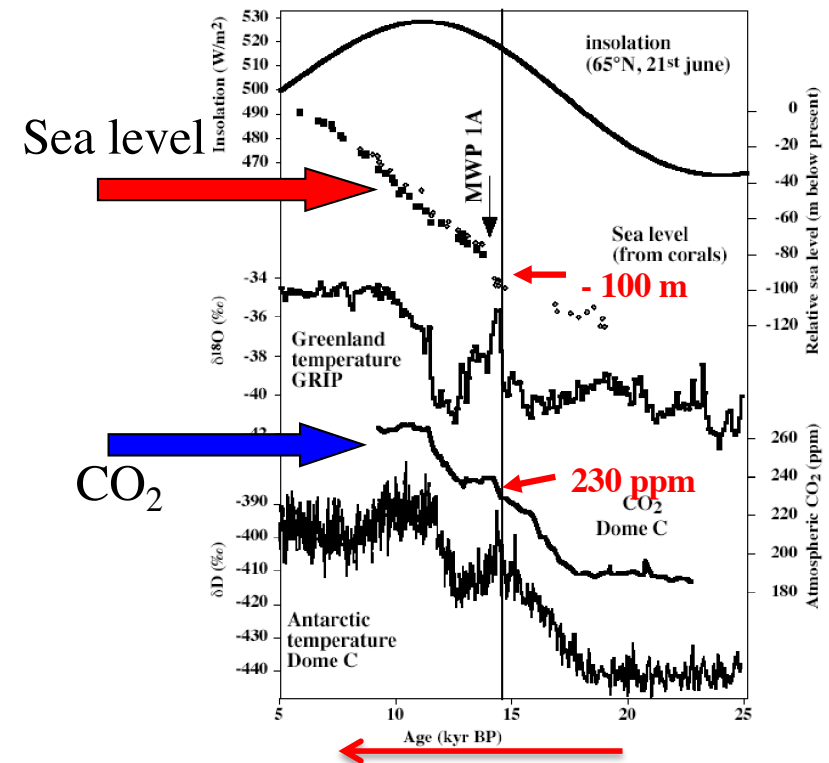
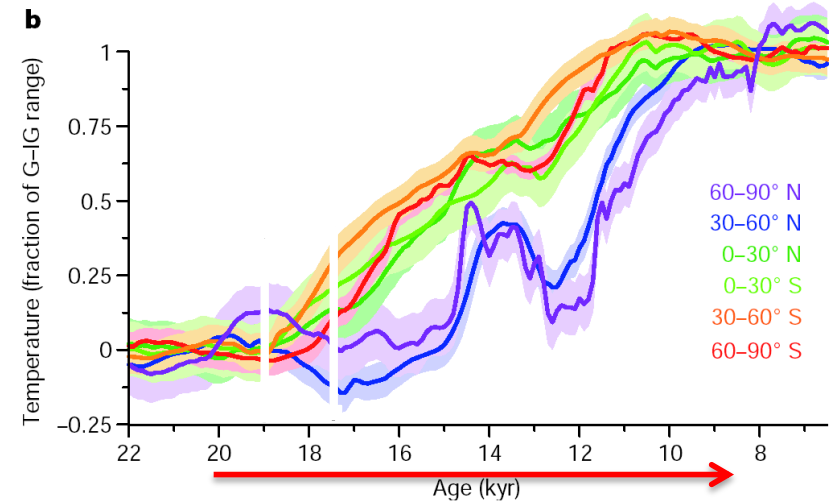
The key (robust) observation:

**CO₂ leads sea-level rise
(deglaciation) by a few millenia**

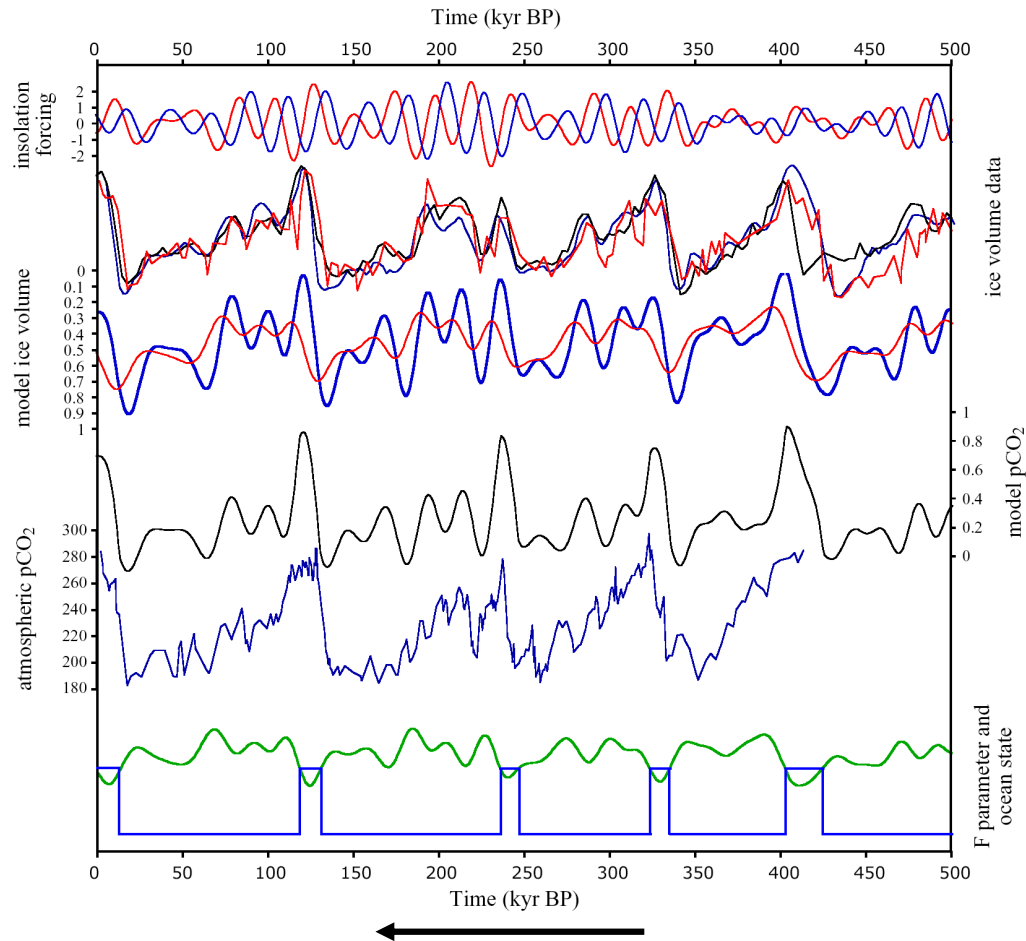


If you posulate that CO₂ has some key
role in the deglaciation:

**CO₂ release is « triggered » by
glacial maxima.**



Conceptual model for ice sheet volume and CO₂



$V = \text{ice volume}$

$A = \text{Antarctic ice sheet area}$

$C = \text{atmospheric CO}_2$

$$\mathbf{F} = \mathbf{a V} - \mathbf{b A} - \mathbf{c I}_{60} + \mathbf{d}$$

$$\mathbf{C}_R = \alpha \mathbf{I}_{65} - \beta \mathbf{V} + \gamma \mathbf{H}(-\mathbf{F}) + \delta$$

$$\mathbf{V}_R = -\mathbf{x C} - \mathbf{y I}_{65} + \mathbf{z}$$

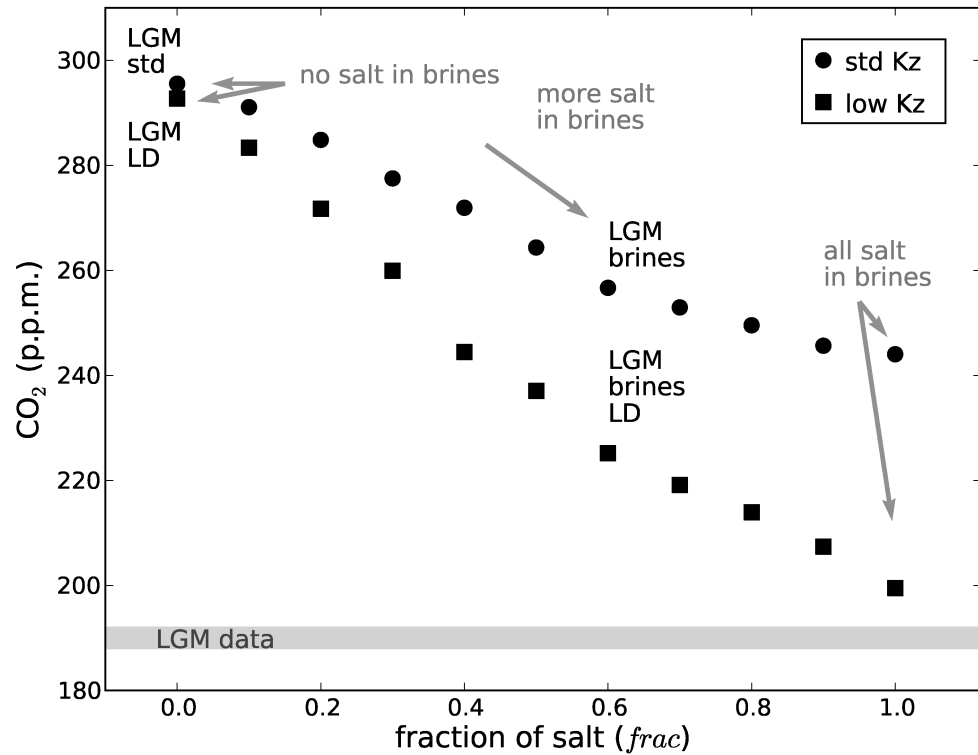
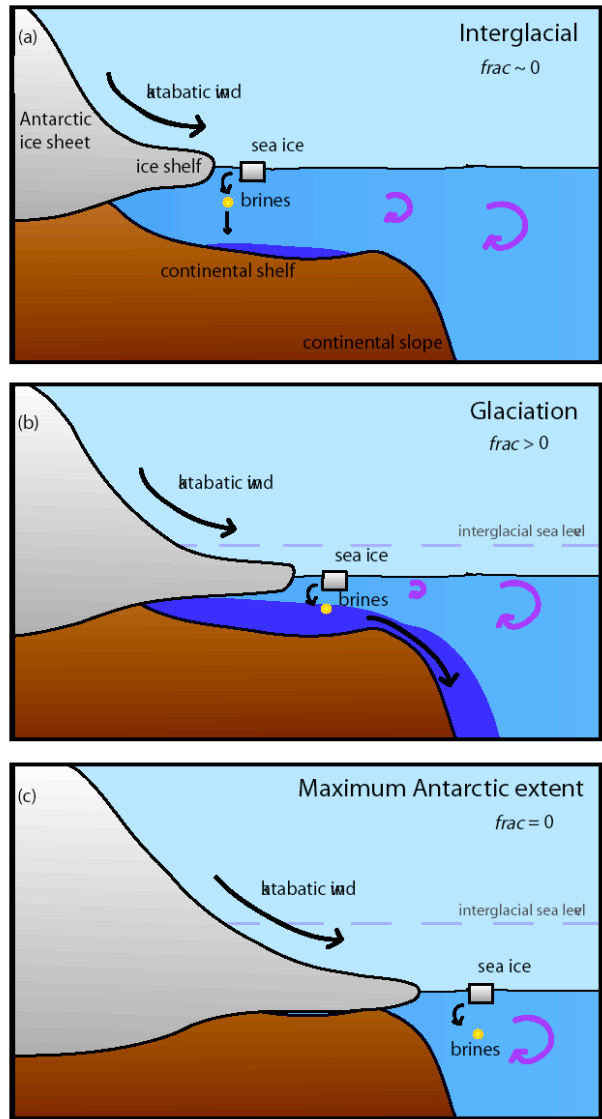
$$\frac{dV}{dt} = \frac{V_R - V}{\tau_V}$$

$$\frac{dA}{dt} = \frac{V - A}{\tau_A}$$

$$\frac{dC}{dt} = \frac{C_R - C}{\tau_C}$$

(Paillard and Parrenin, EPSL, 2004)

The triggering mechanism : brines sinking



(Bouttes et al. 2010)

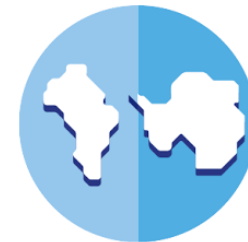
Tipping points and Earth's history

1 – Introduction

- Earth's history : a succession of tipping points
- What is climate ?
- Climate surprises (IPCC 2001 – DOs and HEs)

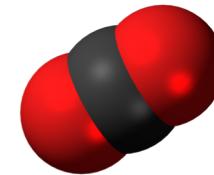
2 – Ice ages and astronomy

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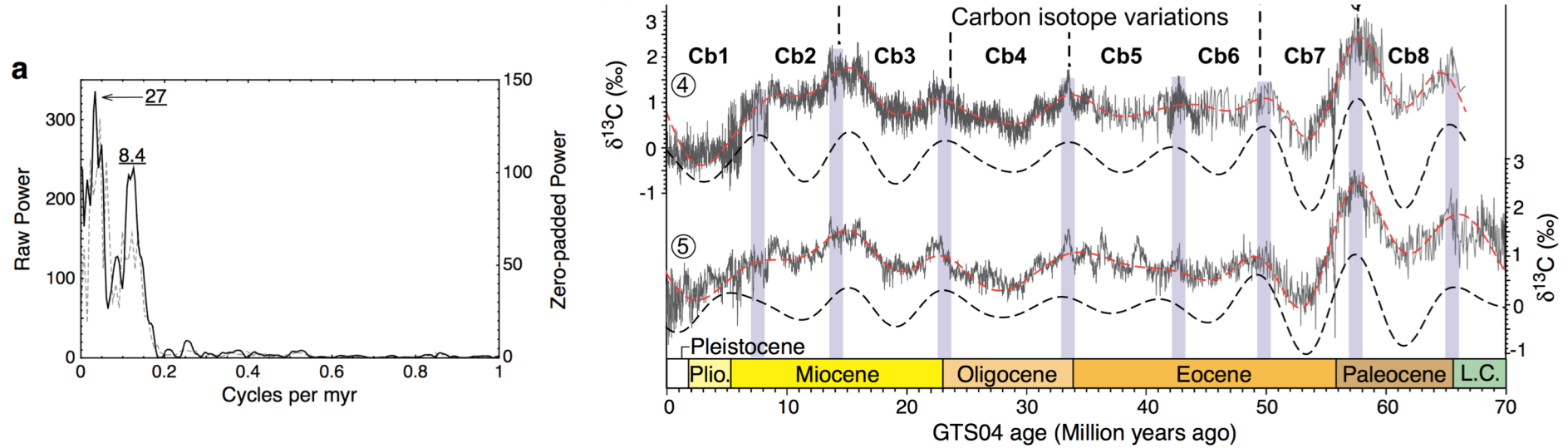
- Some data
- A new theory (involving a tipping point!)



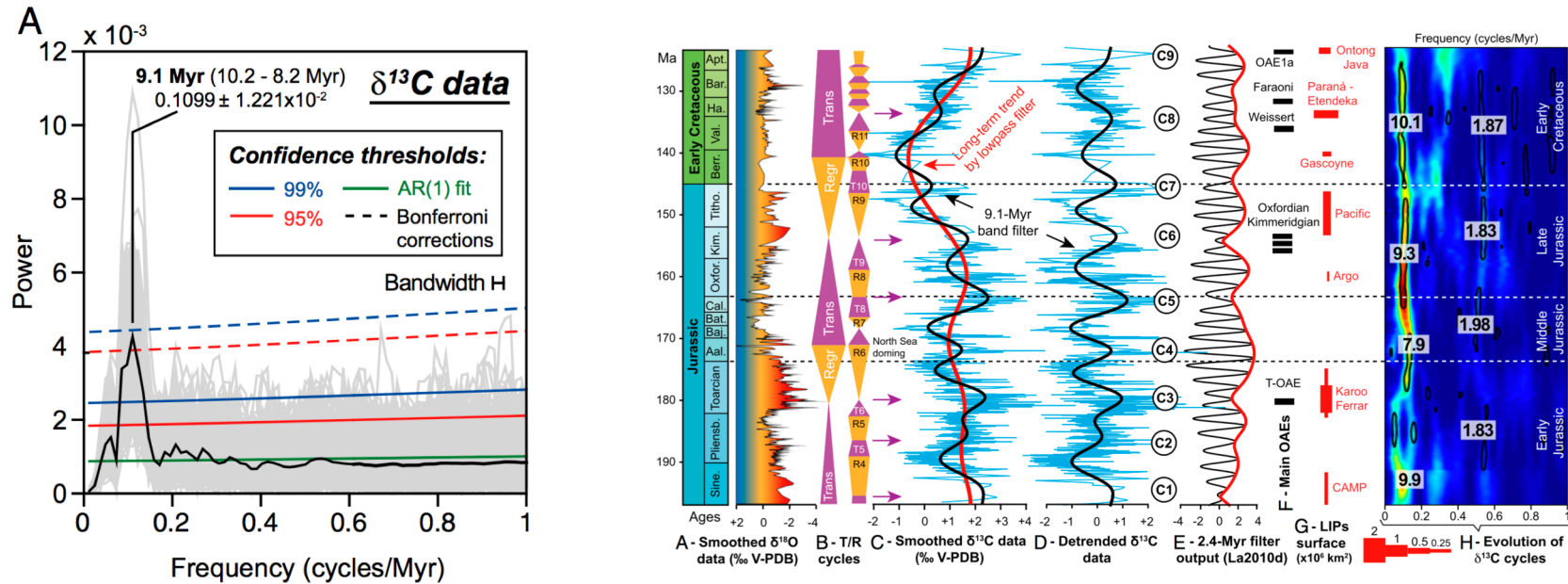
4 – Concluding remarks

What about ^{13}C 9-Ma cycles ?

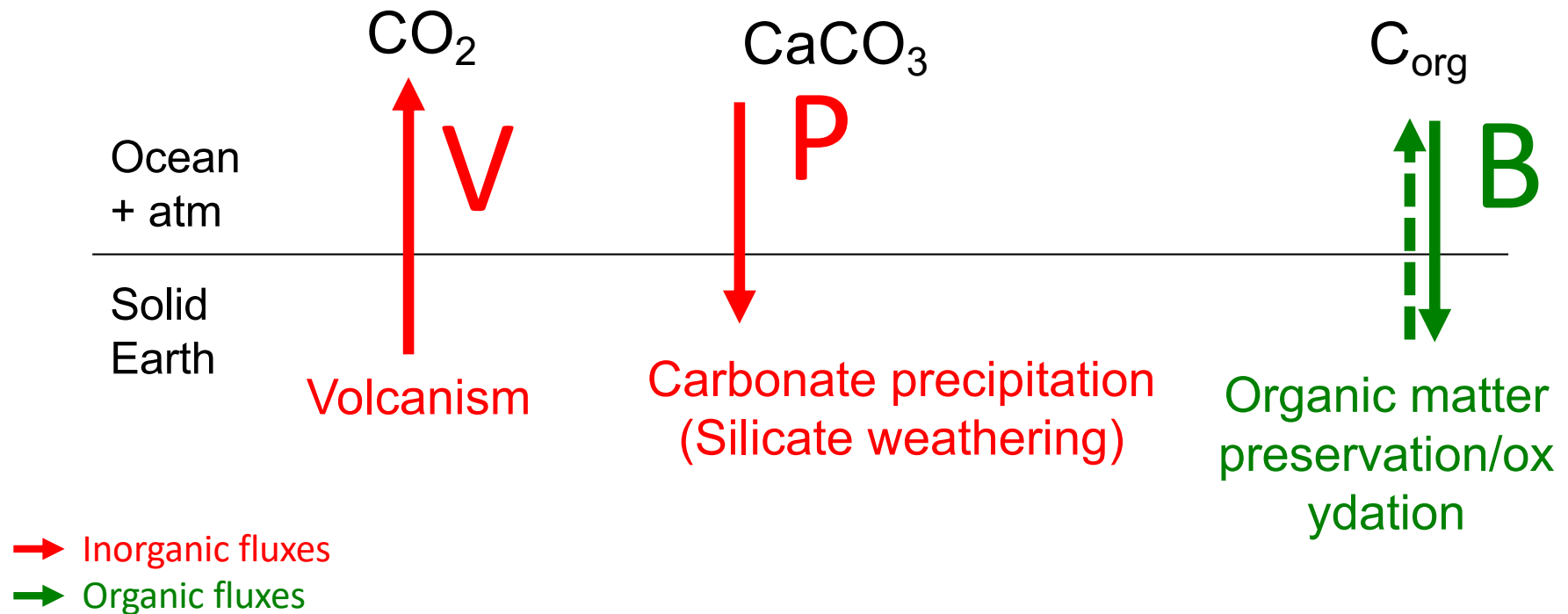
During the Cenozoic (Boulliliah et al. 2012)



During the Mesozoic (Martinez et al. 2015)



Simple view of Earth's carbon

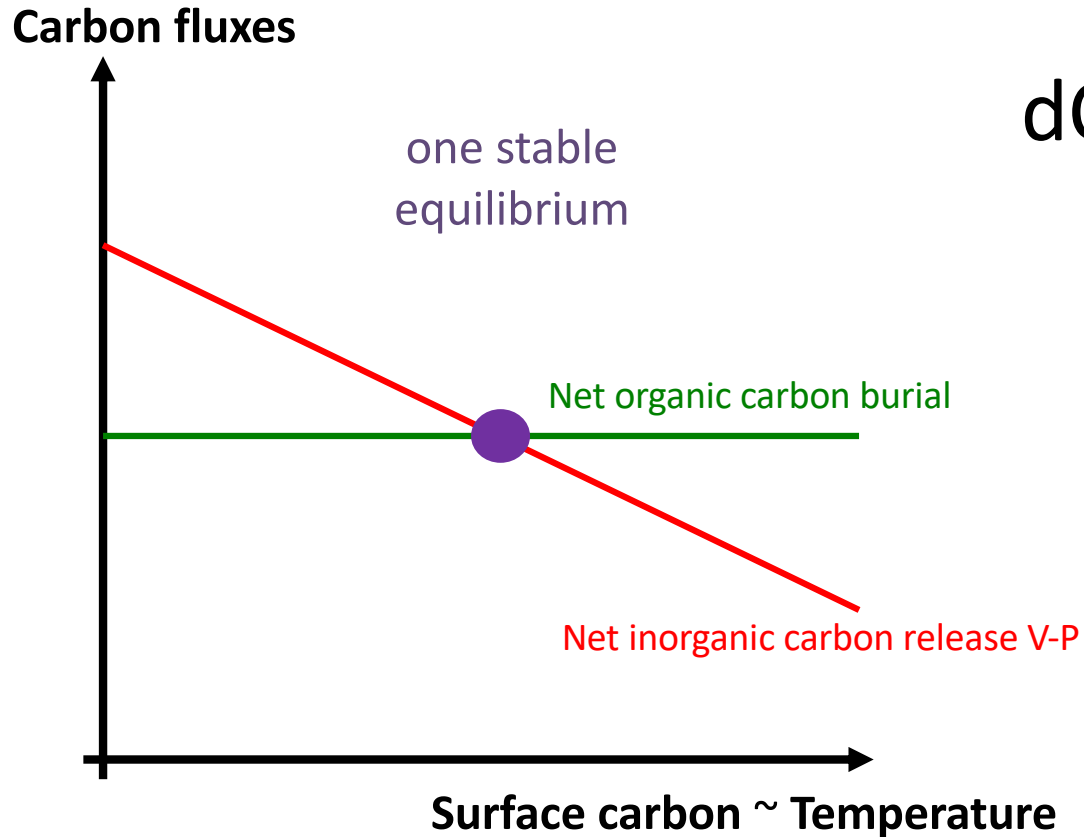


$$dC/dt = V - P - B$$

$$d(C\delta^{13}C)/dt = (V\delta^{13}C_V) - (P\delta^{13}C) - (B\delta^{13}C_B)$$

Simple view of Earth's carbon

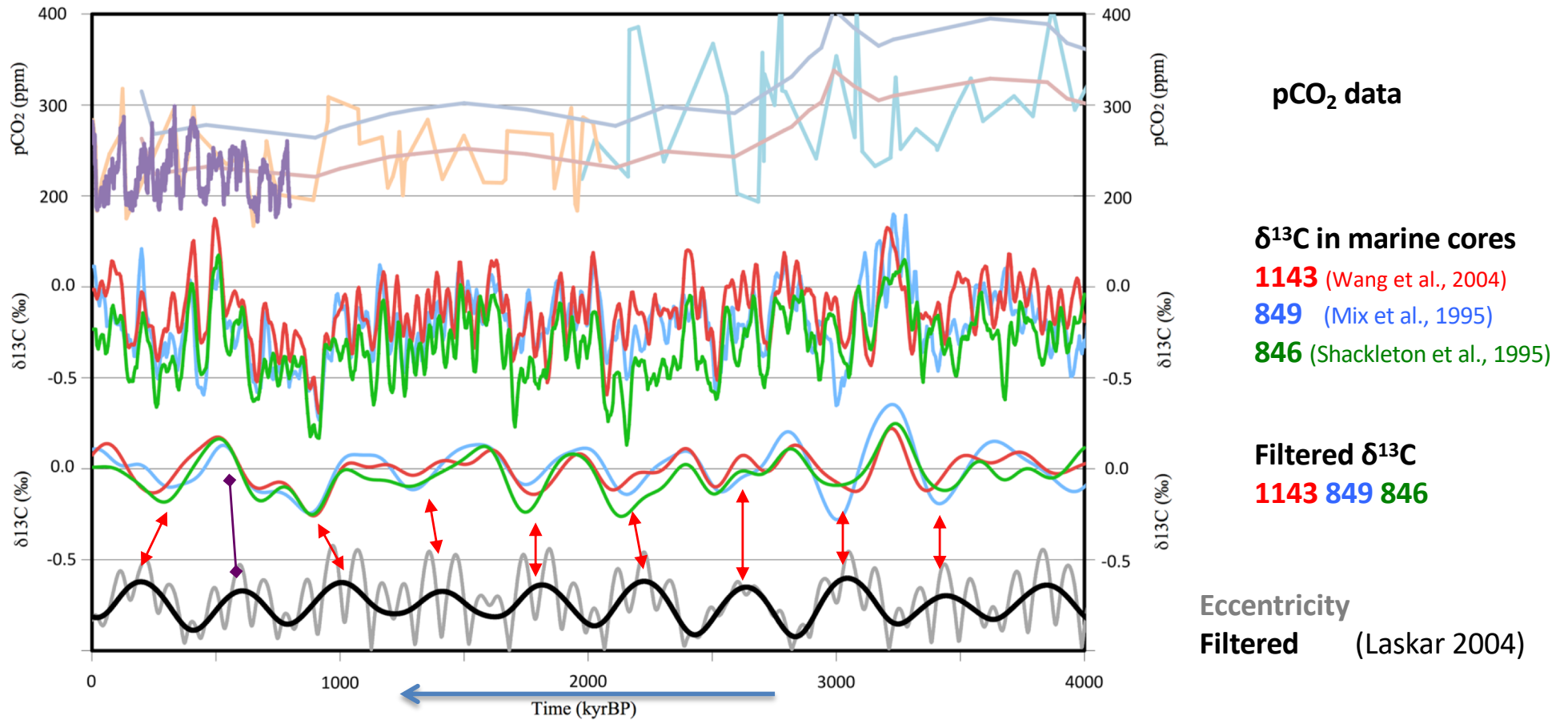
Silicate weathering is an increasing function of carbon (and climate)



$$dC/dt = V - P(T(C)) - B$$

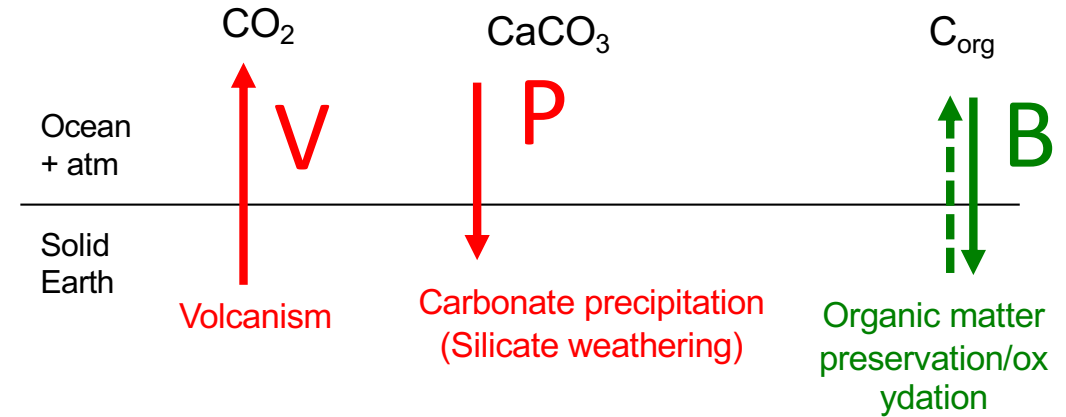
« The Earth's thermostat (Walker et al, 1981) »

^{13}C 400-ka cycles over the last few million-years



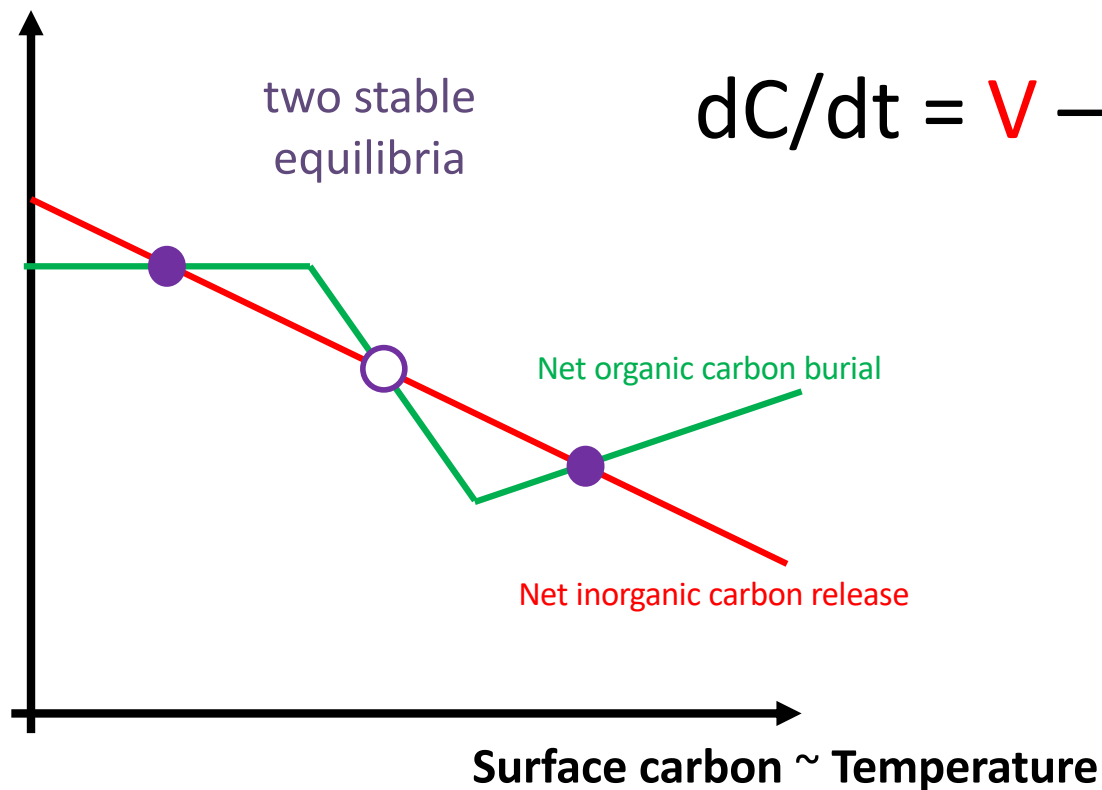
(Paillard, CP, 2017)

^{13}C about 9-Ma cycles from eccentricity ?

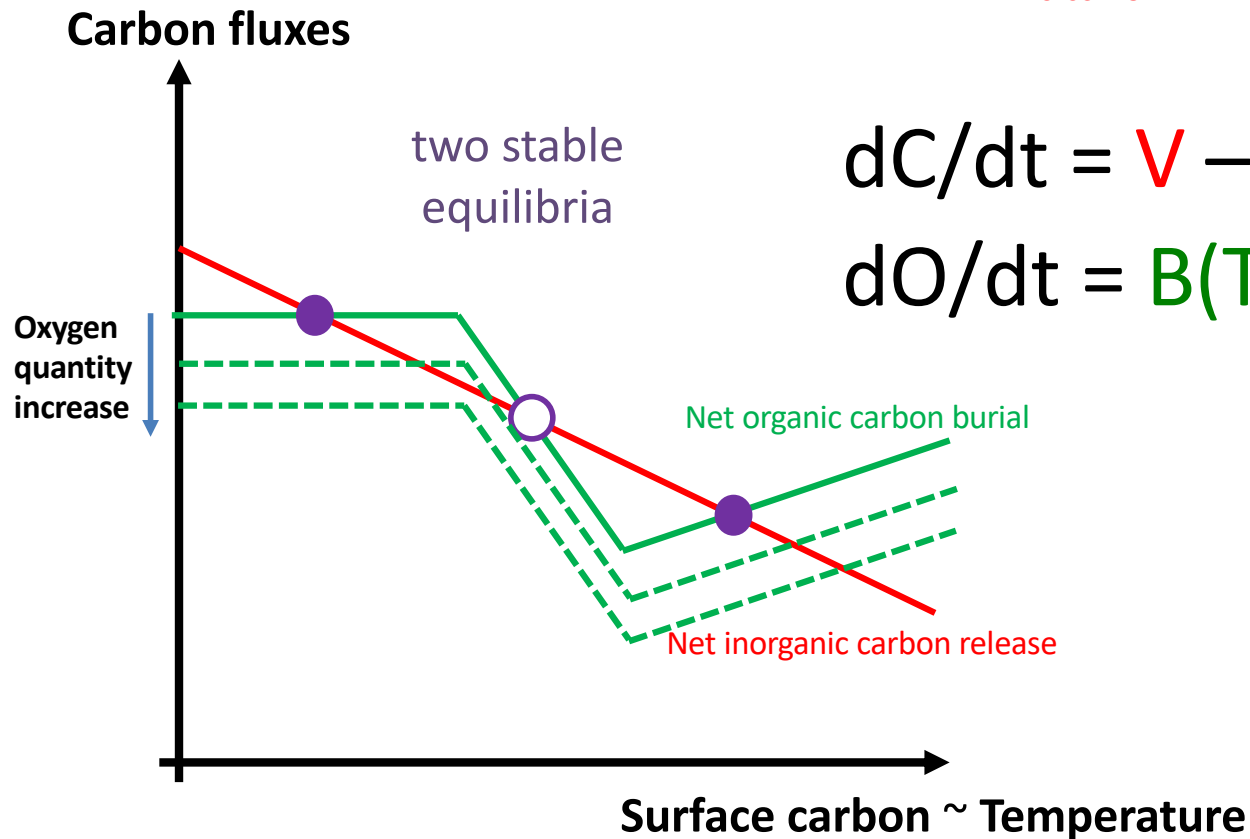
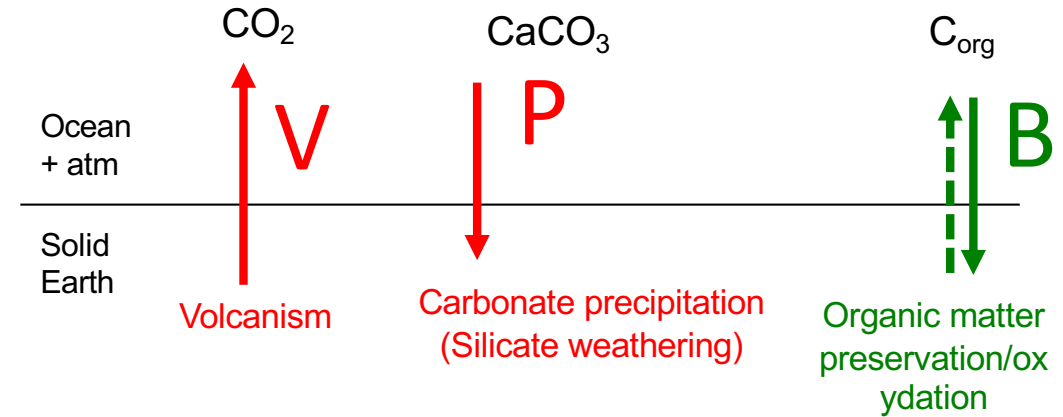


$$dC/dt = V - P(T(C)) - B(T(C))$$

Carbon fluxes



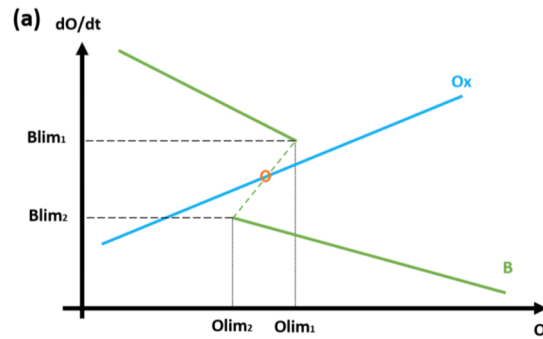
^{13}C about 9-Ma cycles from eccentricity ?



$$dC/dt = V - P(T(C)) - B(T(C), O)$$

$$dO/dt = B(T(C), O) - Ox$$

^{13}C about 9-Ma cycles from eccentricity ?



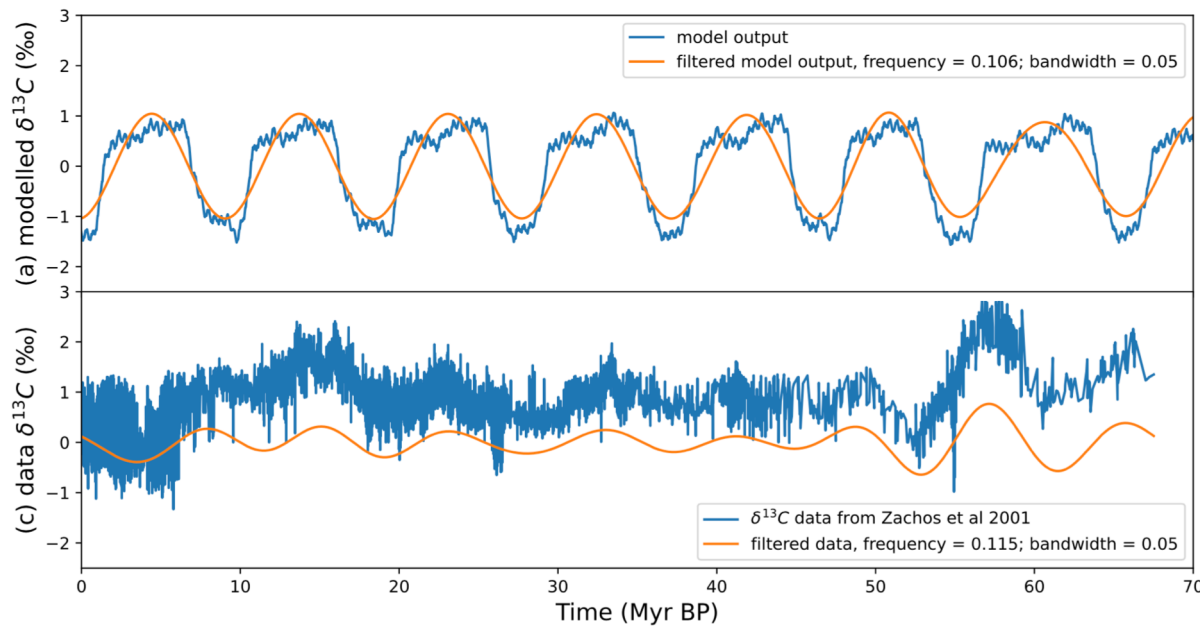
$$dC/dt = 2(V - B) - W$$

$$d\delta^{13}\text{C}/dt = [V(\delta^{13}\text{V} - \delta^{13}\text{C}) - B(\delta^{13}\text{B} - \delta^{13}\text{C})]/C$$

$$dO/dt = B - Ox$$

$$B = B_0(C, O) - a F(t)$$

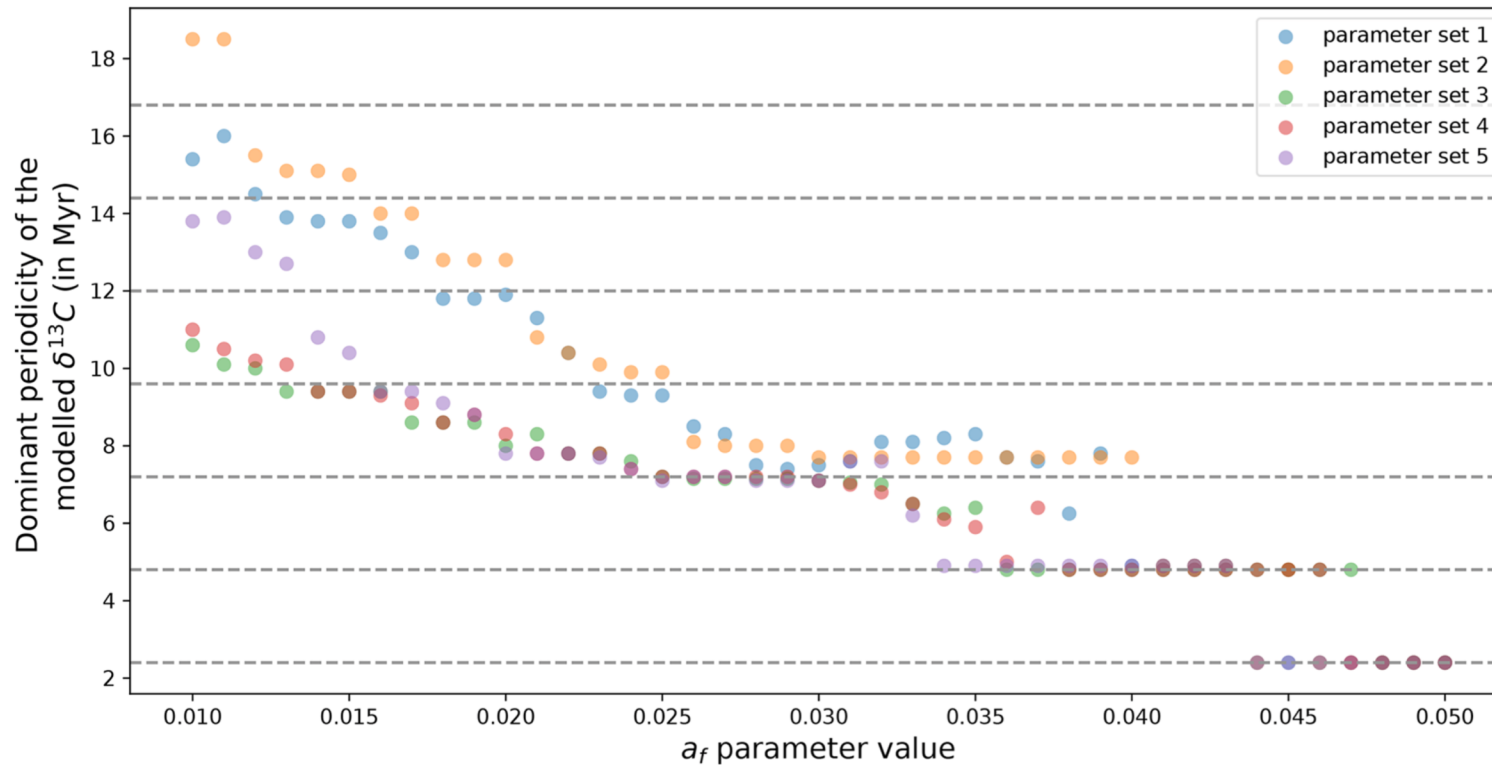
$$F(t) = \max(0, -e \sin \omega)$$



(Leloup & Paillard, ESD, 2023)

^{13}C about 9-Ma cycles from eccentricity ?

Phase locking to eccentricity



Thresholds and hysteresis in the long-term carbon cycle can possibly explain the observed 9-Myr cycles, as a multiple of the eccentricity cycles

(Leloup & Paillard, ESD, 2023)

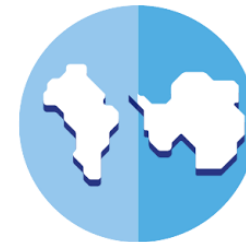
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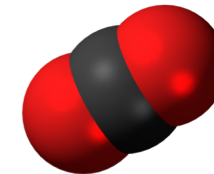
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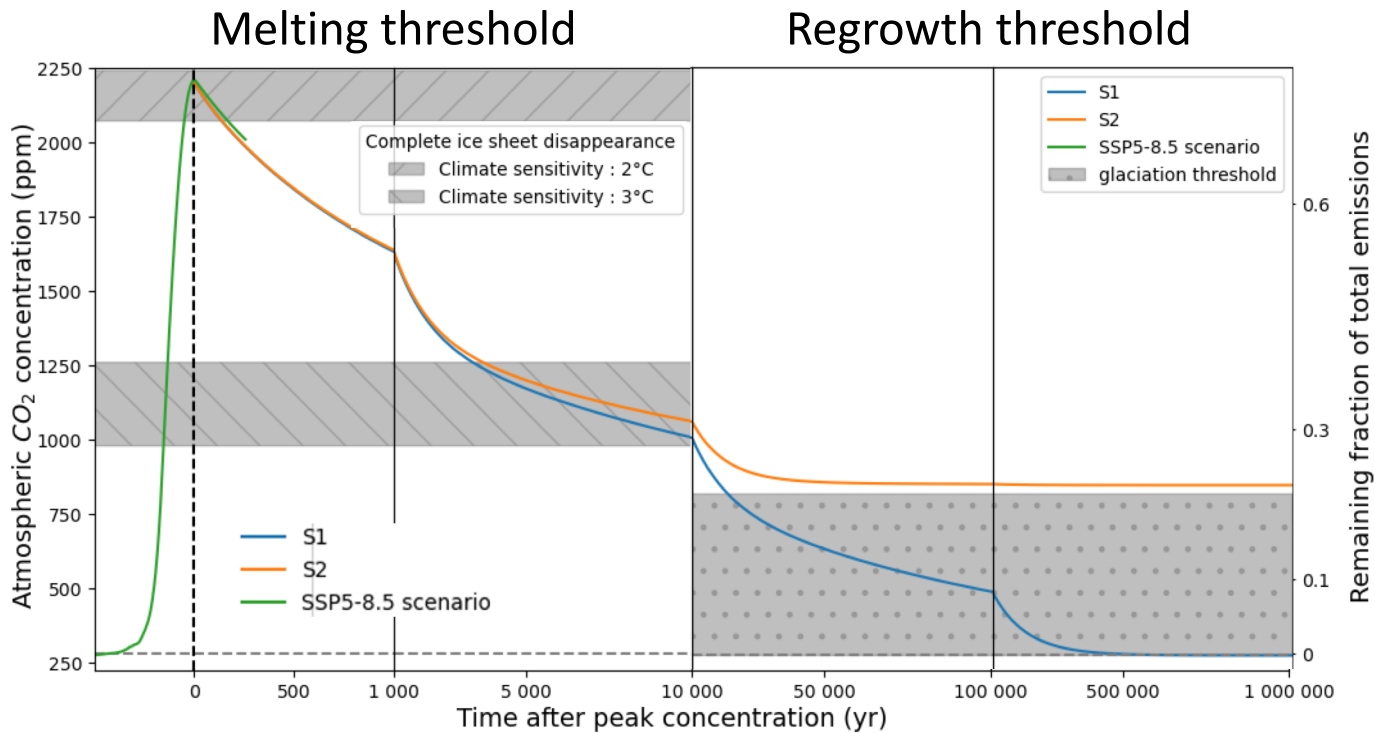
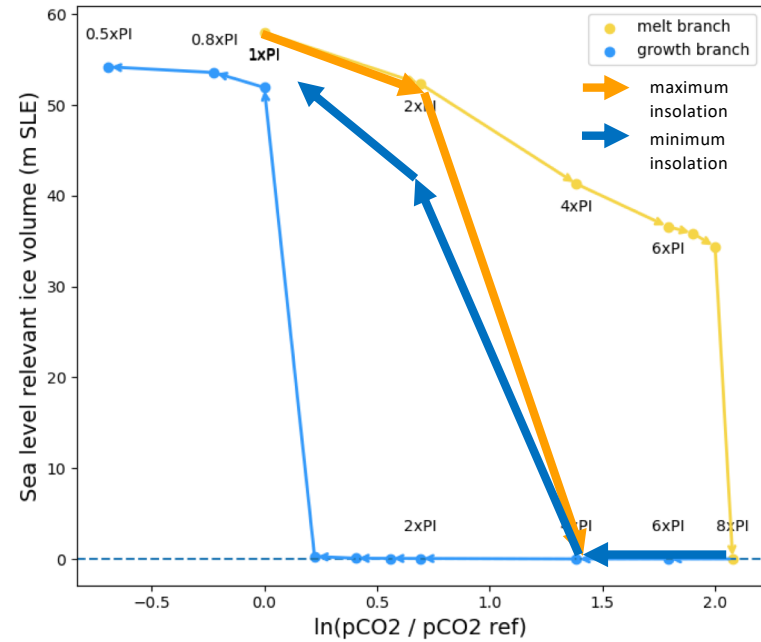
- Some data
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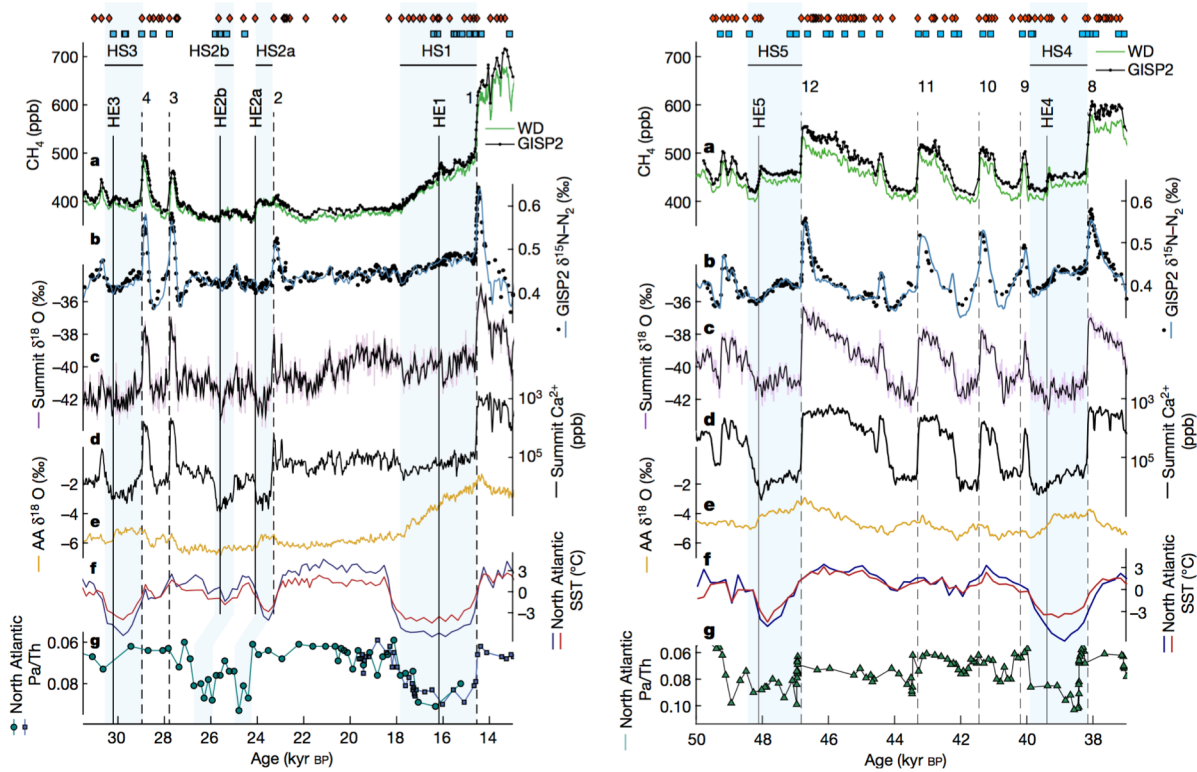
4 – Concluding remarks

Long-term future and fate of Antarctica ?

Discussing the ultimate fate of Antarctica from a coupled climate ice-sheet model (iLoveClim-Grisli) using the equilibrium hysteresis

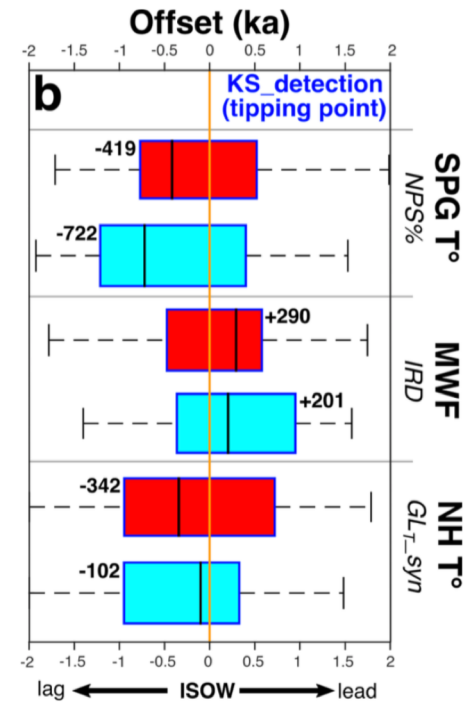


Fresh water fluxes : a consequence not a cause !

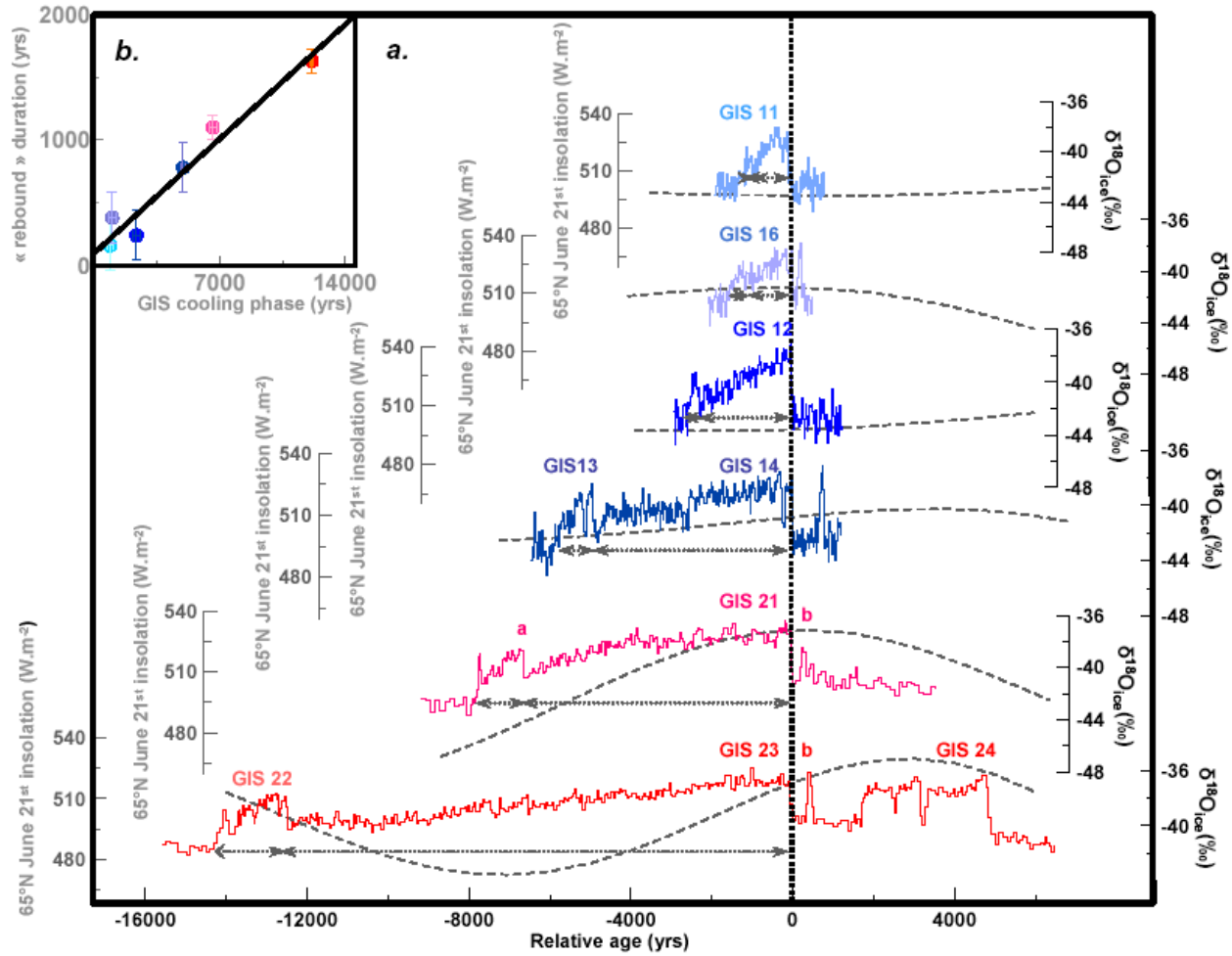


(Martin et al, Nature 2023)

(Stevenard, PhD thesis 2023)

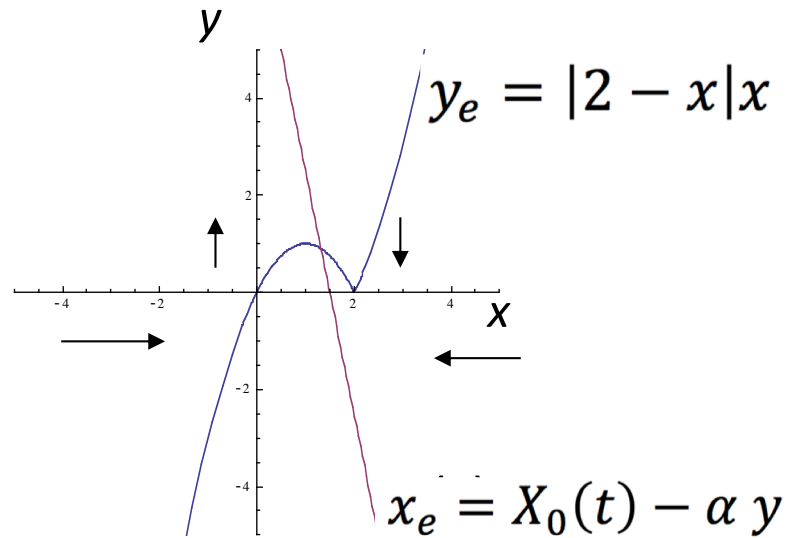


A strong structure of DO events



Capron et al. CP (2010)

Conceptual models to discuss mechanisms

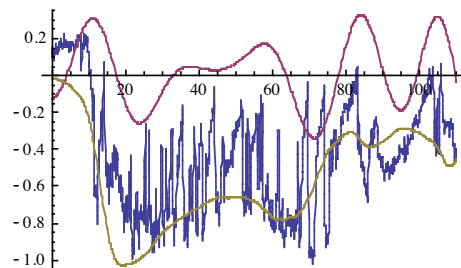
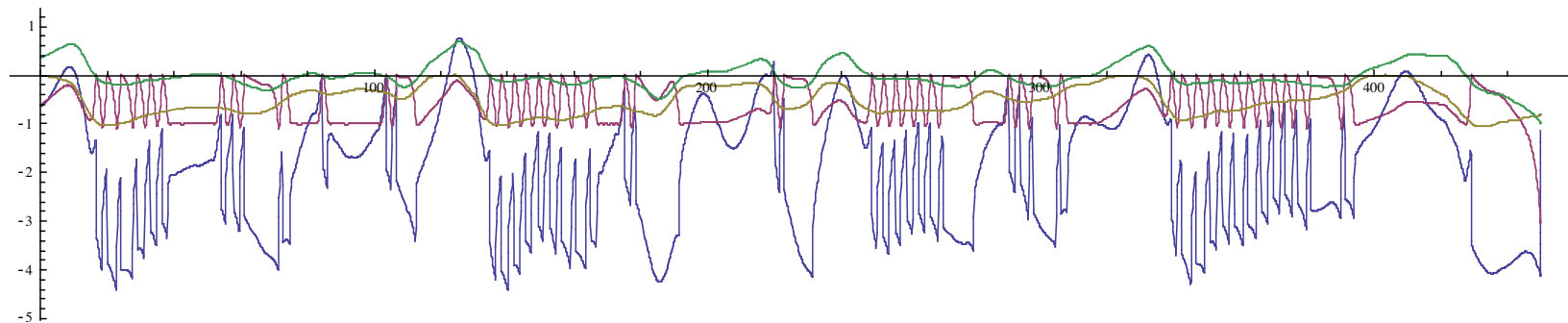


$$\frac{dx}{dt} = \frac{1}{\tau_x} (y - y_e)$$

$$\frac{dy}{dt} = -\frac{1}{\tau_y} (x - x_e)$$

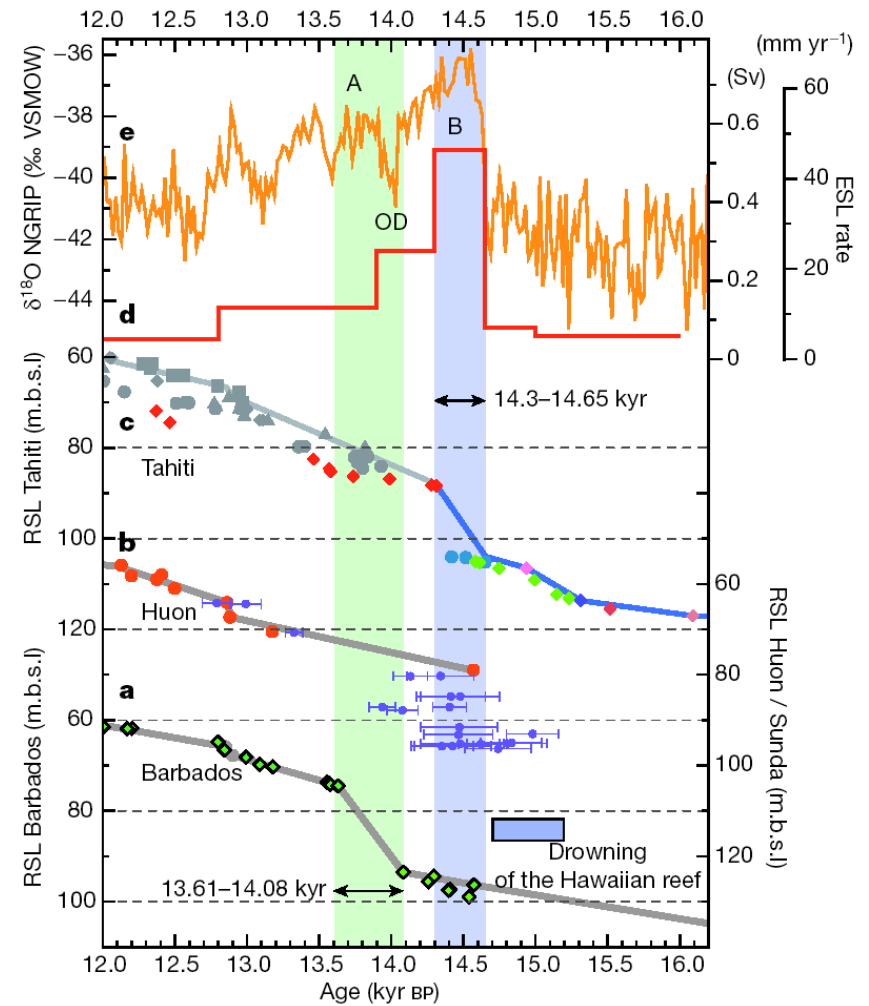
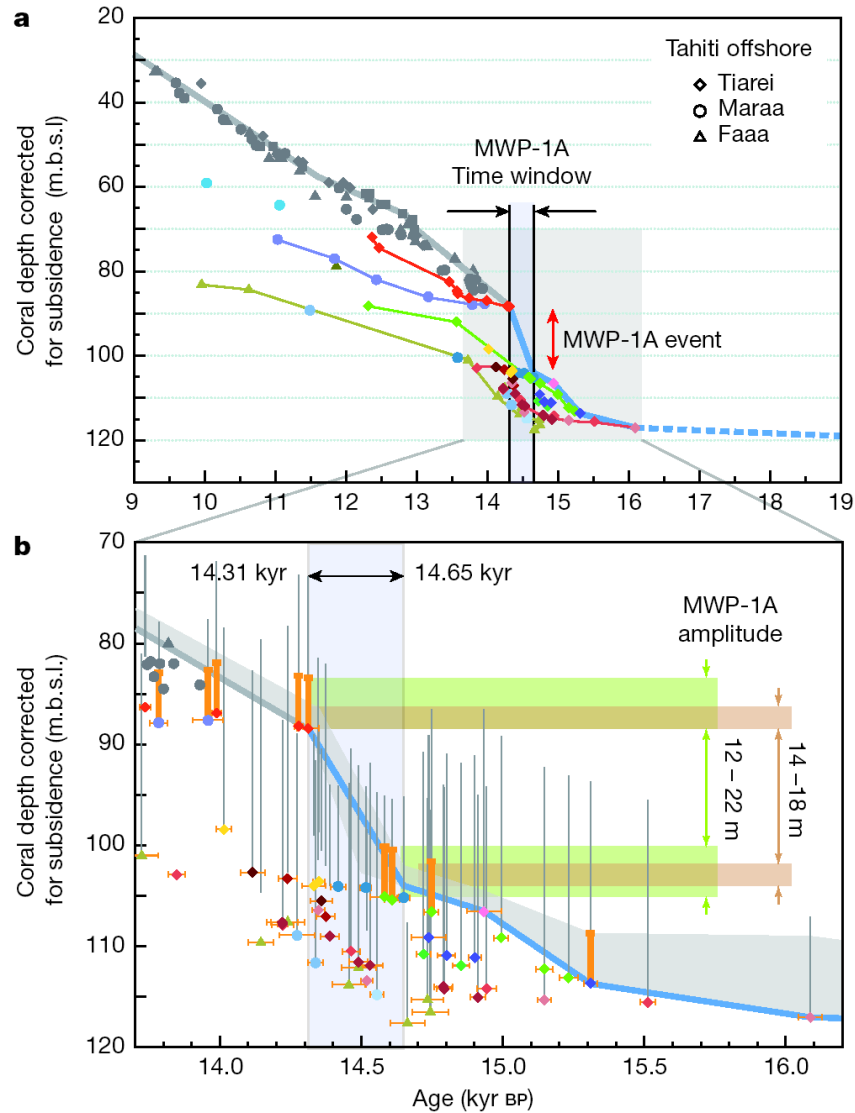
$$\frac{da}{dt} = \frac{1}{\tau_a} (sl(t) - a)$$

$$X_0(t) = x_0 + \beta(a - sl(t))$$



Thank you !

L'énigme du « Meltwater Pulse 1A » ~ 4 à 5 m / siècle



(Deschamps et al., Nature 2012)

The ice age problem

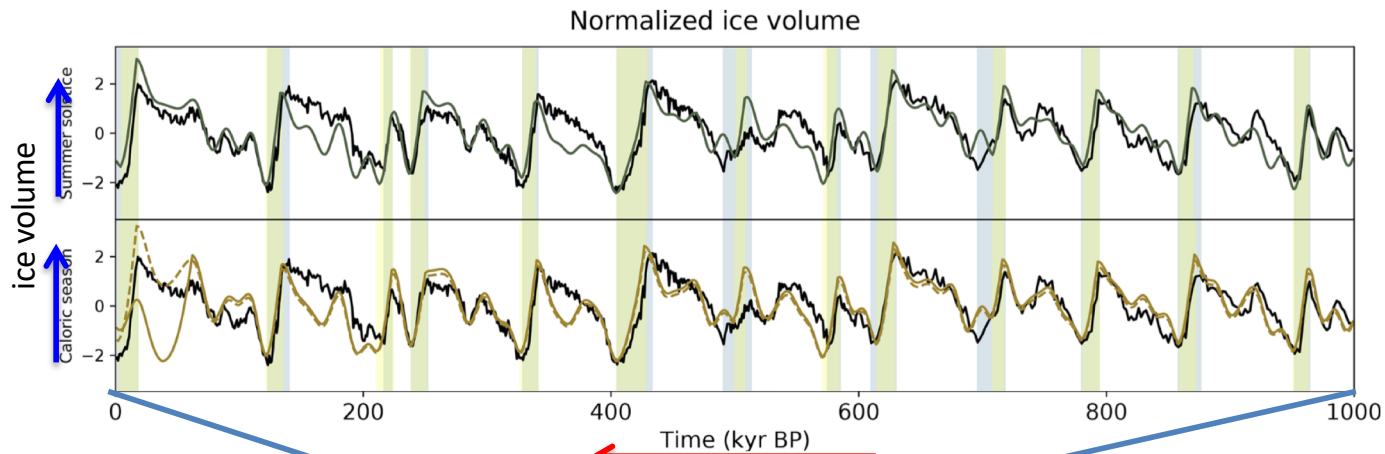
Simple linear dynamics...

$$\begin{cases} (g) \frac{dv}{dt} = -\frac{I}{\tau_i} + \frac{1}{\tau_g} \\ (d) \frac{dv}{dt} = -\frac{I}{\tau_i} - \frac{v}{\tau_d} \end{cases}$$

but with « tipping points »:

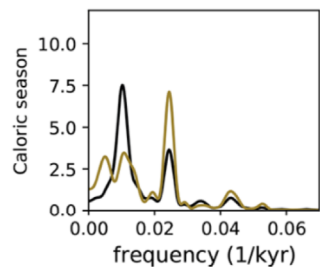
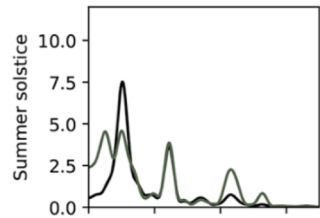
$$\begin{cases} (d) \text{ to } (g) : I < I_0 \\ (g) \text{ to } (d) : I + v > V_0 \end{cases}$$

(relaxation oscillator)

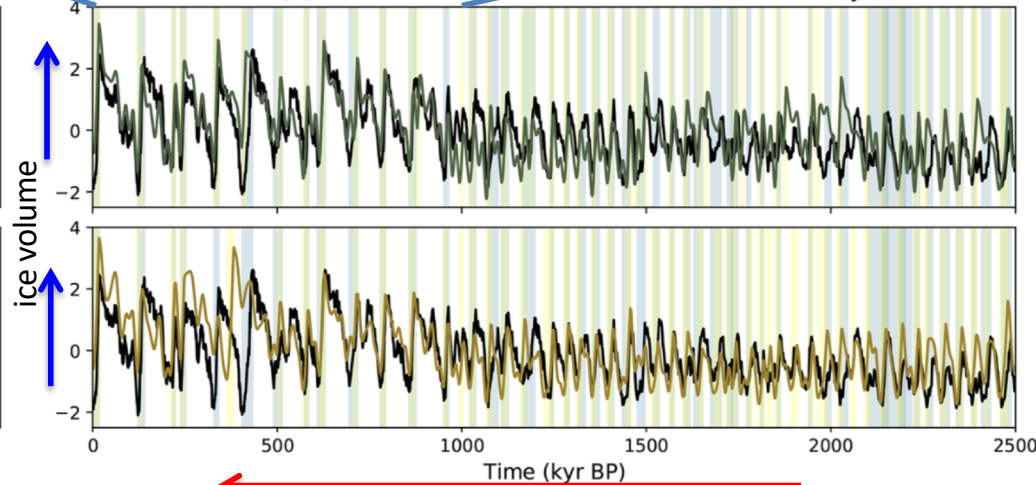


(Leloup & Paillard, 2022)

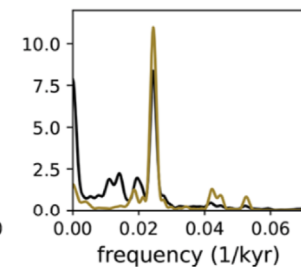
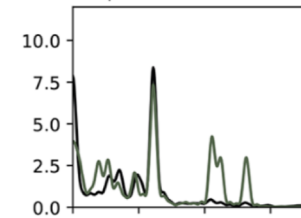
(a) Spectral analysis after the MPT



(b) Normalized ice volume over the Quaternary



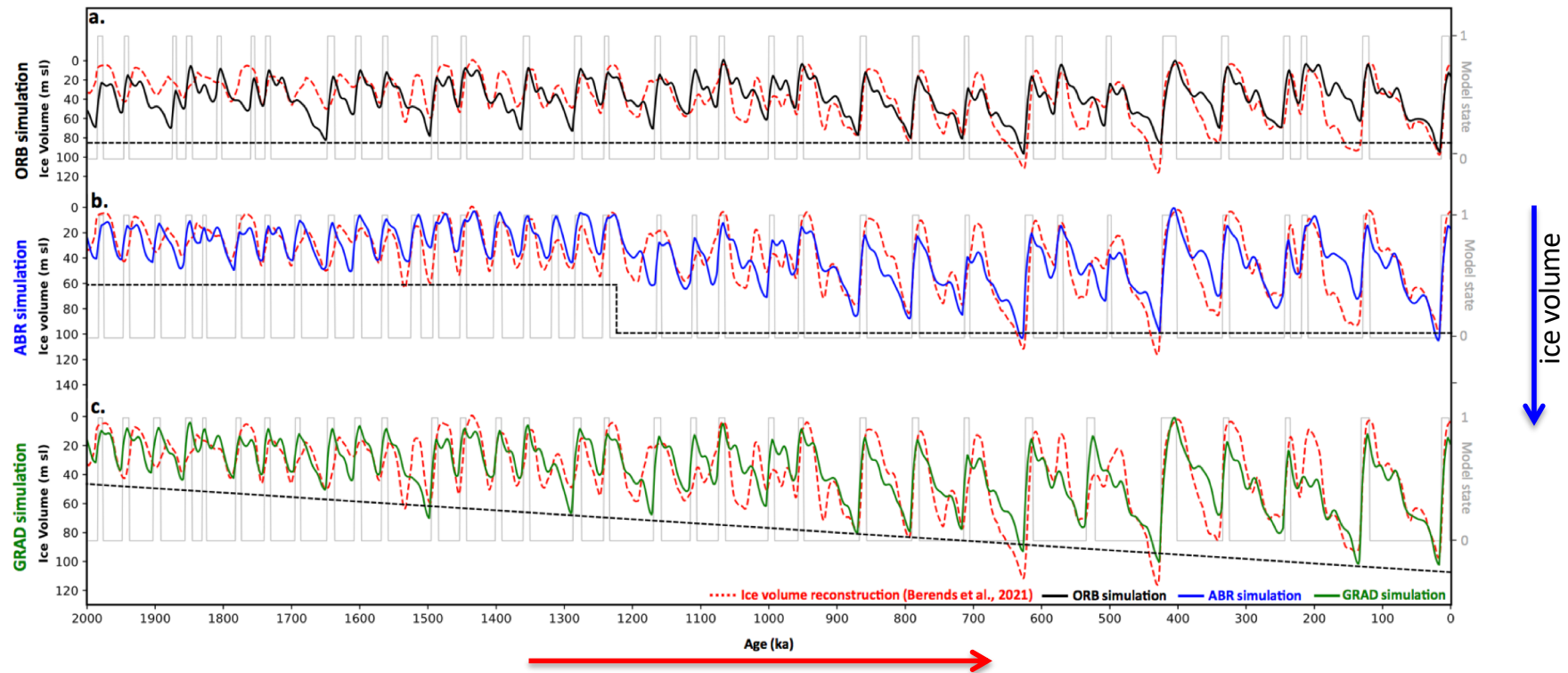
(c) Spectral analysis prior to the MPT



The MPT is likely caused by a gradual change

Legrain et al., Communications Earth & Environment, 2023

Best-fit « linear model with a hysteresis »:



Δ BIC criteria

	Vs. ORB simulation	Vs. ABR simulation	Vs. GRAD simulation
ORB simulation	0	-	-
ABR simulation	49.6	0	-
GRAD simulation	61.7	12.1	0

The ice age problem

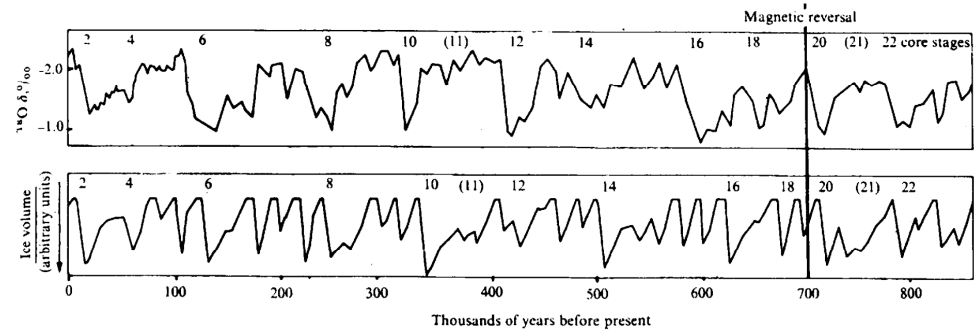
Calder (Nature 1974)

$$\frac{dV}{dt} = -k(i(t) - i_0)$$

$$V(t) \geq 0$$

$$k = \begin{cases} k_M, & \text{if } i(t) > i_0 \\ k_A, & \text{if } i(t) < i_0 \end{cases}$$

$$\begin{cases} i_0 = 502 \text{ W.m}^{-2} \\ \frac{k_A}{k_M} = 0,22 \end{cases}$$

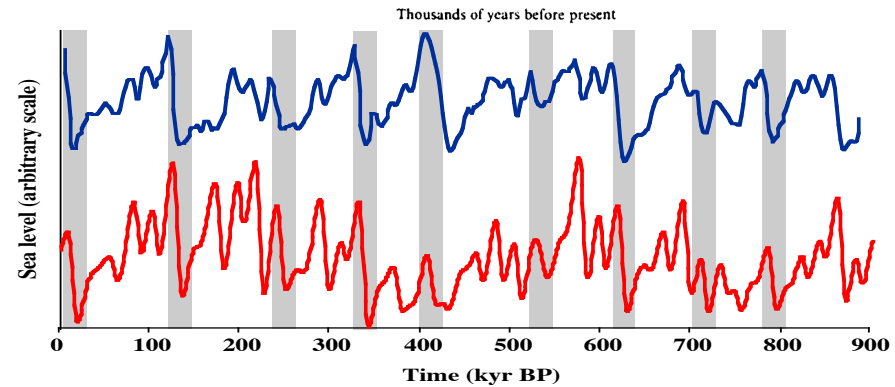


Imbrie and Imbrie (Science 1980)

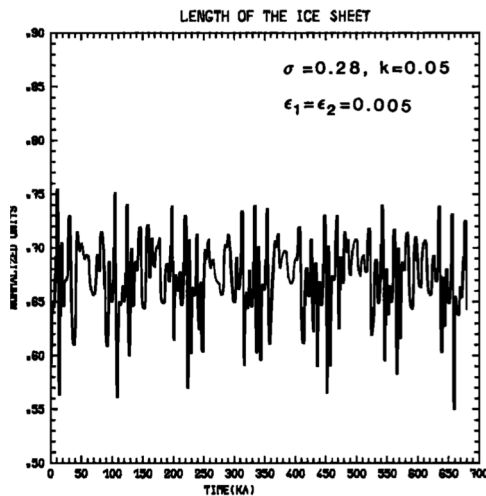
$$\frac{dV}{dt} = \frac{V_R - V}{\tau}$$

$$V_R = -i(t)$$

$$\tau = \begin{cases} \tau_M = 10 \text{ kyr} & \text{if } V > V_R \ (\dot{V} < 0) \\ \tau_A = 42 \text{ kyr} & \text{if } V < V_R \ (\dot{V} > 0) \end{cases}$$



Le Treut and Ghil (JGR 1983)



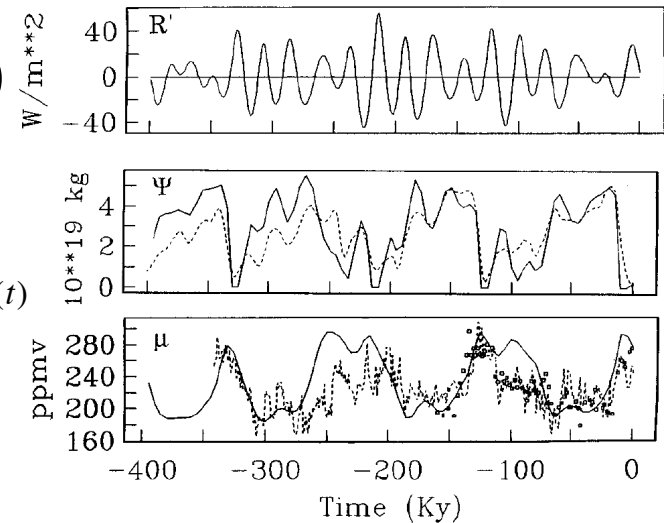
Saltzman's models (1977; ...2001)

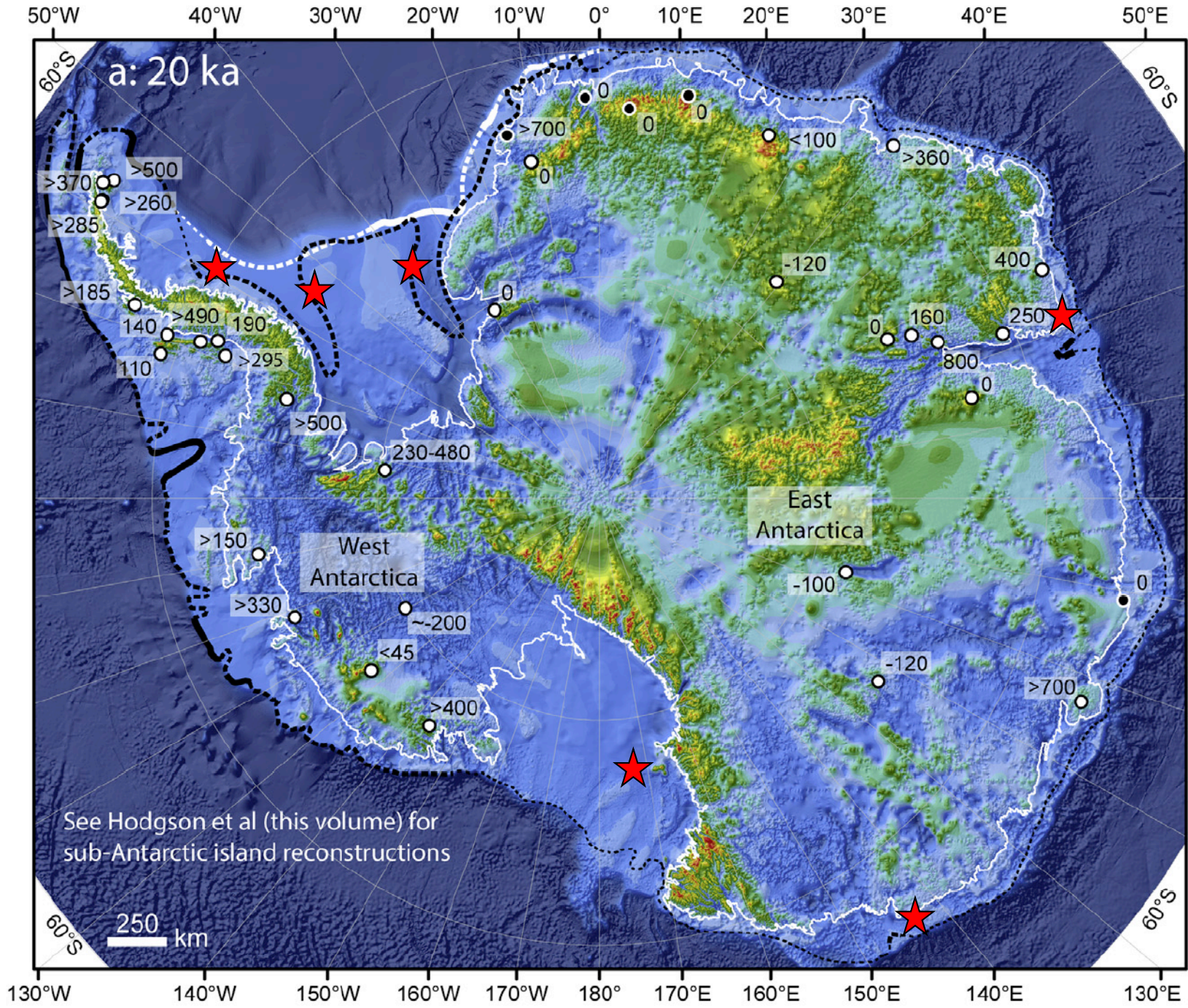
$$\frac{dV}{dt} = \alpha_1 - \alpha_2(cC + k_1\theta + k_2F_V(t)) - \alpha_3V$$

$$\frac{dC}{dt} = \beta_1 - (\beta_2 - \beta_3C + \beta_4C^2)C - \beta_5\theta + F_C(t)$$

$$\frac{d\theta}{dt} = \gamma_1 - \gamma_2V - \gamma_3\theta$$

Saltzman et al. (1993)





The RAISED consortium et al, QSR, 2014)

QUARTERLY JOURNAL
OF THE
ROYAL METEOROLOGICAL SOCIETY

Vol. XXVII.] JANUARY 1901 [No. 117. pp. 1-61.

ON THE VARIATIONS OF THE CLIMATE OF THE
GEOLOGICAL AND HISTORICAL PAST AND THEIR CAUSES.

By DR. NILS EKHOLM, Hon. Mem. Roy. Met. Soc.,
Meteorologiska Central-Anstalten, Stockholm.



We have seen that the *present burning of pit-coal is so great that* in one year it gives back to the atmosphere about 1/1000 of its present store of carbonic acid. If this continues... *it will undoubtedly cause a very obvious rise of the mean temperature of the Earth.*

Further, it might perhaps be possible for Man to diminish or regulate the consumption of carbonic acid by protecting *the weathering layers of silicates from the influence of the air* and by ruling the growth of plants.

Thus it seems possible that Man will be able efficaciously to regulate the future climate of the earth and consequently *prevent the arrival of a new Ice Age*. By such means also the deterioration of the climate of the northern and Arctic regions *depending on the decrease of the obliquity* of the ecliptic may be counteracted.

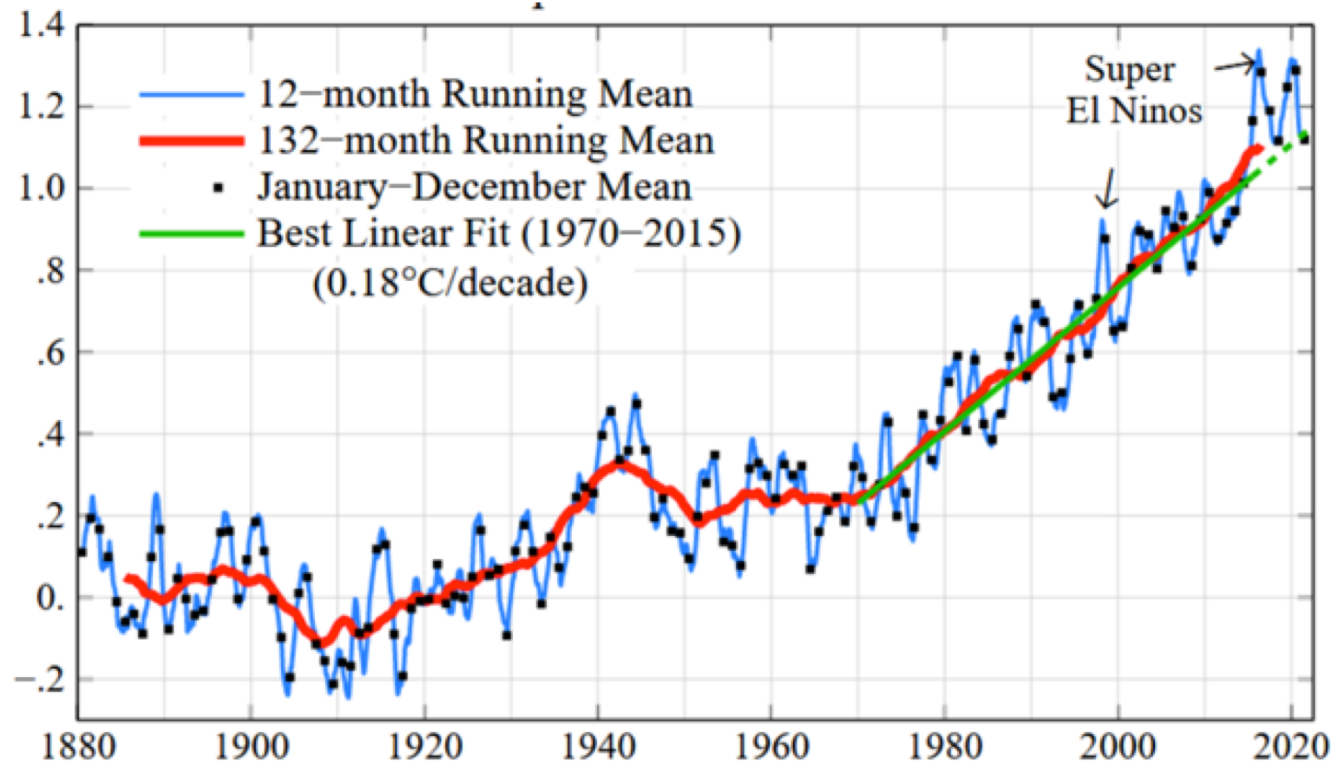
I cannot help thinking that it will afford to *Mankind* hitherto unforeseen means of *evolution*.



Nils Ekholm,

Quarterly Journal of the Royal Meteorological Society,
January 1901

« The present burning of pit-coal is so great that ... it will **undoubtedly cause a very obvious rise** of the mean temperature of the Earth ... »



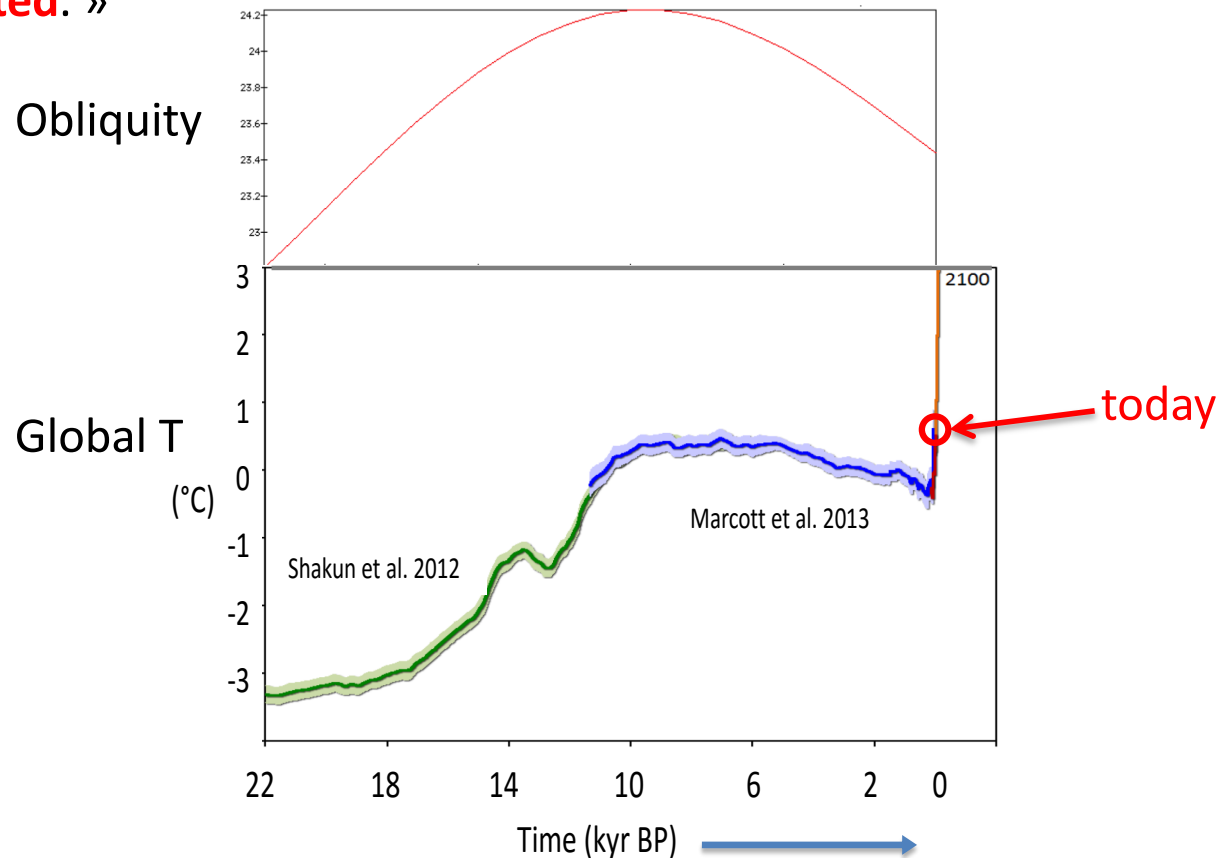


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« By such means also the deterioration of the climate of the northern and Arctic regions, depending on the **decrease of the obliquity, may be counteracted.** »





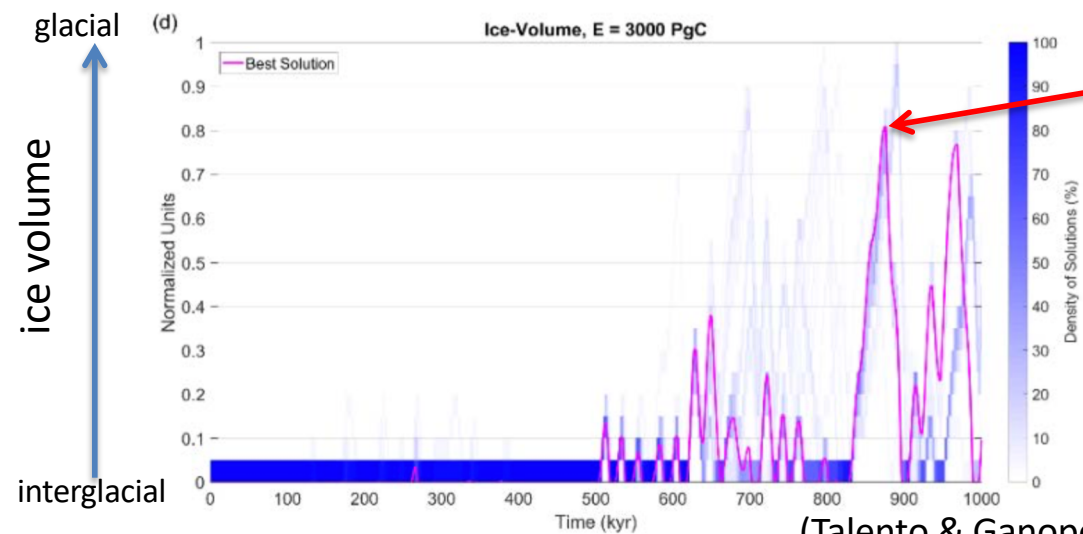
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«Thus it seems possible that Man will be able efficaciously to regulate the future climate of the Earth and consequently **prevent the arrival of a new Ice Age.** »



next ice age
not before
820 000 ans

(Talento & Ganopolski, 2021)

